Development of Empirical Equations For Water-Oil Relative Permeability

by

Saud Mohammed A. Al-Fattah

A Thesis Presented to the

FACULTY OF THE COLLEGE OF GRADUATE STUDIES

KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

DHAHRAN, SAUDI ARABIA

In Partial Fulfillment of the
Requirements for the Degree of

MASTER OF SCIENCE

In

PETROLEUM ENGINEERING

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This thesis, written by **Mr. Saud Mohammed Abdul-Aziz Al-Fattah** under the direction of his Thesis Advisor and approved by his Thesis Committee, has been presented to and accepted by the Dean of the College of Graduate Studies, in partial fulfillment of the requirements for the degree of **MASTER OF SCIENCE** in **Petroleum Engineering.**

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DEDICATION

I dedicate this work to my parents and family, wife and sons

May Allah Almighty Bless them

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I am greatly indebted to Dr. Muhammad A. Al-Marhoun, my thesis major advisor, for his guidance and help throughout the course of this research. I would like also to express my sincere appreciation to the other committee members, Dr. Khalid A. Al-Fossail and Dr. Hasan Y. Al-Yousef, for their valuable suggestions and comments.

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التخـــــصص : هندسة البترول .

تاريخ التـــخرج : ديسمبر ١٩٩٤م .

تعتبر معلومات النفاذية النسبية ضرورية جداً لمعظم حسابات تدفق الموائع في مجال هندسة النفط . حيث تقوم منحنيات النفاذية النسبية للماء والزيت بدوراً هام في تحديد إنسياب الماء والزيت المتزامن في الصخور المسامية وفي التنبؤ بأداء عمليات الإزاحة الغير ممزوجة في مكامن الزيت .

في هذه الدراسة تم تطوير معادلات رياضية جديدة لحساب النفاذية النسبية المشبّعة للصخور الرملية في وجود الماء والزيت وذلك بإستخدام ٨٢٧ معلومة تجريبية مستخلصة من ٤٦ إختبار إزاحي لعينات لبّيه لصخور رملية . ولقد أشتقت النماذج الرياضية للنفاذية النسبية بدلالة كل من خواص الصخور وخواص الموائع بإستخدام طرق تحاليل الإحصاء إلانحساريه الخطية وغير الخطية . هذا ولقد تم مقارنة المعادلات الرياضية الجديدة بالمعادلات الرياضية السابقة في هذا المجال وذلك بإستخدام بيانات مخبرية أستخدمت في هذه الدراسة وأيضاً بإستخدام بيانات مخبرية أخرى منشورة . حيث برهنت النتائج الإحصائية أن المعادلات الرياضية المطورة في هذا البحث تفوق في الأداء جميع المعادلات الرياضية السابقة في كلى حالتي إستخدام المعلومات الداخلة في مرحلة التطوير والتقويم .

درجة ماجستير العلوم

جامعة الملك فهد للبترول والمعادن

الظهران - المملكة العربية السعودية

ديسمبر ١٩٩٤م

THESIS ABSTRACT

Full Name of Student	:	Saud Mohammed A. Al-Fattah
Title of Study	:	Development of Empirical Equations for Water-Oil Relative Permeability.
Major Field	:	Petroleum Engineering
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Relative permeability data are essential for almost all calculations of fluid flow in petroleum engineering. Water-oil relative permeability curves play important roles in characterizing the simultaneous two-phase flow in porous rocks and predicting the performance of immiscible displacement processes in oil reservoirs. New empirical equations were developed in this study to calculate water-oil imbibition relative permeability curves for sandstones based on a total of 827 experimentally obtained data points from 46 displacement core tests. The relative permeability models were derived as a function of rock and fluid properties using stepwise linear and nonlinear regression analyses. The new empirical equations were both evaluated using the data utilized in the development and validated using published data, against previously published equations. Statistical results show that the empirical equations developed in this study outperform all previous empirical equations, for both the data used in the development and validation.

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CHAPTER 1

CHAPTER 1

INTRODUCTION

Fluid flow through porous media is complex. Darcy's law, which forms the basis for almost all calculations of fluid transport through reservoir rocks, relates the macroscopic velocity of a fluid of known viscosity to the pressure gradient by a proportionality factor called absolute permeability. The permeability is defined as the capacity of porous medium to transmit fluid. The absolute permeability is a measurement of the specific flow capacity of a rock saturated 100 percent with a single fluid. Generally, displacement of oil from a porous system by water or gas involves simultaneous flow of two or more immiscible phases. In such conditions, Darcy's law can be generalized to describe the simultaneous flow of more than one fluid by introducing the concept of effective permeability. The effective permeability is a relative measure of the conductance of the porous medium for one fluid phase when the medium is saturated with more than one fluid. It is, therefore, a function of the prevailing fluid saturation, the rock-wetting characteristics, and the geometry of the pores of the rock. Because the presence of two or more immiscible fluids in

a system causes one fluid to interfere with the flow of another, the effective permeability of a certain fluid is always less than its absolute permeability. Effective permeabilities are generally measured in the laboratory on small core samples. However, owing to many possible different saturations for a single medium, laboratory data are usually summarized and reported as relative permeability.

Relative permeability is an important concept in describing the flow of multiphase systems. It is defined as the ratio of the effective permeability of a fluid at a given saturation to the absolute permeability of the rock. Three different base permeabilities are used: (1) the absolute permeability, (2) the absolute water permeability, and (3) the effective permeability to oil at residual wetting-phase saturation. The third base has become common in the industry, and consequently, the relative permeability to oil at connate water saturation is unity. This concept is believed to be more nearly representative to the reservoir than one in which 100 percent saturation of the flowing fluid is required.

Relative permeability is usually plotted as a function of fluid saturation. Figure 1.1 is a typical water-oil relative permeability curve. It is helpful to plot such curve on a semi-log scale to reveal the relative permeability characteristics near the endpoint saturations. It is also not uncommon to plot the relative permeability ratios vs. saturation on a semi-log scale to aid in the analysis of the relative permeability characteristics.

Relative permeability is a function of pore geometry, wettability, fluid distribution, and saturation history. The relative permeability of a phase usually depends on the path that was followed to reach the saturation. This path dependency, termed hysteresis, is analogous to the variation of capillary with

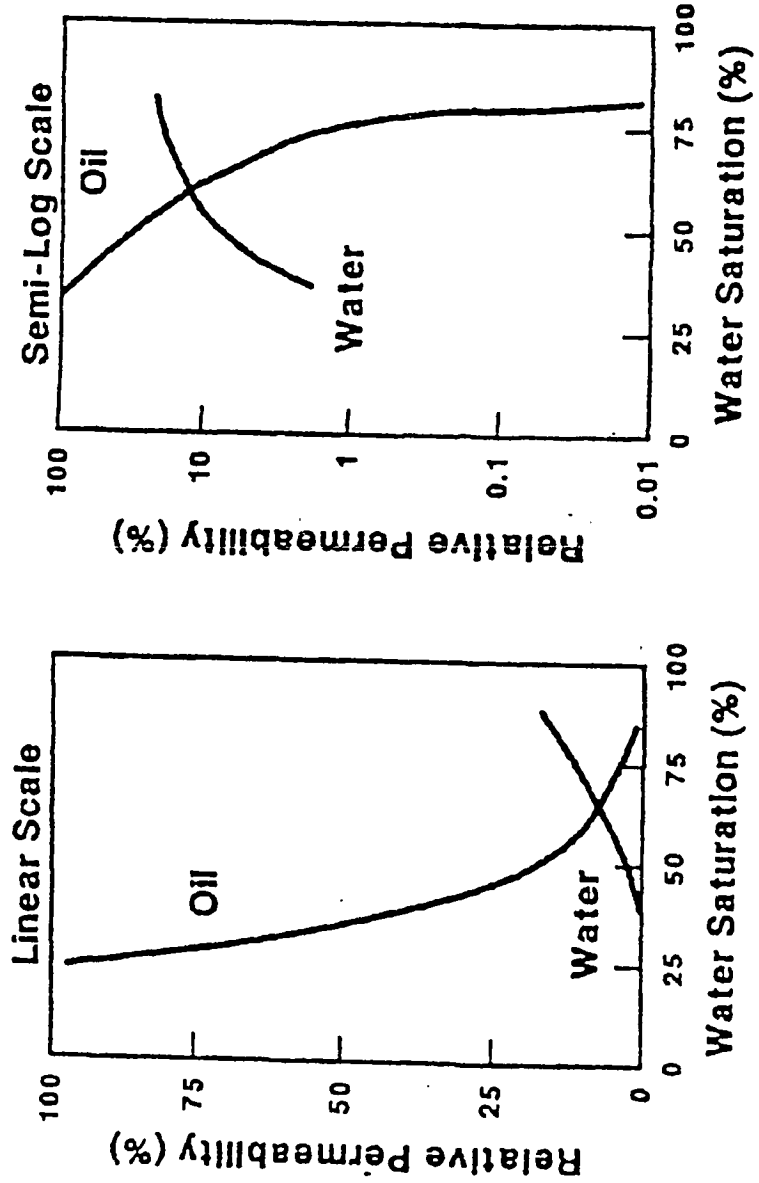


Fig. 1.1: Typical Water-Oil Relative Permeability Curves.

the direction of saturation change. Because of the saturation history affects the fluid distribution and causes a hysteresis in the capillary pressure curves, a similar hysteresis in the drainage and imbibition relative permeability curves can be expected. If the rock sample is initially saturated with the wetting phase and relative permeability data are obtained by decreasing this phase saturation by a nonwetting phase, the process is called a drainage. The imbibition process is the reverse of a drainage process; the data are obtained by increasing the saturation of the wetting phase.

Relative permeability data are essential for almost all calculations of fluid flow in petroleum reservoirs. The data are used in making engineering estimates of productivity, injectivity, and ultimate recovery. Some applications of relative permeability data include determination of free water surface, aid in evaluating drill-stem and production tests, determination of residual fluid saturations, fractional flow and frontal advance calculations to determine the fluid distributions, and making future predictions for all types of oil reservoir under different operational schemes. Undoubtedly, these data are considered probably the most valuable information required in reservoir simulation studies.

Estimates of relative permeabilities are generally obtained from laboratory experiments with reservoir core samples using one of the measurement methods: steady-state or unsteady-state techniques. The relative permeability data may also be determined from field data using the production history of a reservoir and its fluid properties. However, this approach is not often recommended because it requires complete production history data and provides average values influenced by pressure and saturation gradients, differences in the depletion stage, and saturation variations in stratified

reservoirs. In addition, the agreement between laboratory determined relative permeabilities and those calculated from production history is generally poor. Because the laboratory measurement of relative permeabilities is rather delicate and time consuming, empirical correlations are usually employed to reproduce experimentally determined relative permeability curves, or to estimate them when experimental data from core samples are not available.

The purpose of this study is to develop new empirical equations to accurately predict imbibition water-oil relative permeability characteristics using experimentally obtained data for sandstone reservoir rocks. Multiple linear and nonlinear least-square regression techniques are applied on the new proposed models utilizing the experimental rock and fluid saturation data to generate the coefficients of the empirical equations. The new empirical equations will be evaluated against several empirical equations published in the literature using the data used in the development, and will be validated using published relative permeability data.

CHAPTER 2

CHAPTER 2

LITERATURE REVIEW

Empirical models for determining relative permeability are becoming more applicable today, particularly with the rapid evolution of complex reservoir simulators. This is due to the difficulties involved in the direct experimental measurement of relative permeability from core samples in the laboratory. Empirical models are based on proposed empirical equations describing experimentally determined relative permeabilities. Empirical equations are usually used to reproduce experimentally determined relative permeability curves, or to predict them when experimental data are unavailable. The next section reviews several published empirical equations for calculating water-oil relative permeability curves. The following highlights the results of the early studies on calculating relative permeabilities based on theoretical and analytical models. The equations that form these models can be found in related literature[6-55].

In 1927, Kozeny-Carman[17] published an equation as one of the early attempts to relate several laboratory measured-parameters to rock

permeability. This equation relates the permeability of a porous medium to the product of the effective path length of the flowing fluid and the average hydraulic radius of the channels through which the fluid flows.

In 1949, Purcell[39] proposed another equation for calculating the permeability of a porous medium as a function of the porosity and the capillary pressure curve of that medium. In formulating the equation, Purcell considered the porous medium as a bundle of capillary tubes of different sizes.

In 1949, Rose[43] developed a useful method for computing the relative permeability of wetting and nonwetting phases based on an analysis of the physical interrelationship between the fluid flow phenomena in porous media and the static and residual saturation values. The principal drawback of Rose's method is that the residual saturation of both wetting and nonwetting phases must be known fairly accurately.

The equations developed by Kozeny-Carman and Purcell were modified by several investigators[6,11,13,40,55] for the calculation of relative permeability. The modified equations were based on the assumption that a porous medium consists of a bundle of capillary tubes in order to apply Darcy's and Poiseuille's equations in the derivations.

In 1950, Gates and Lietz[13] developed an equation based on Purcell's model for calculating the wetting phase relative permeability of California cores.

In 1951, Rapoport and Leas[40] presented two equations for determining the relative permeability to the wetting phase based on surface energy relationships and Kozeny-Carman equation. The equations were

presented as defining limits for the maximum and minimum wetting-phase relative permeability.

In 1951, Fatt and Dykstra[11] published an equation for computing the relative permeability following the basic technique of Purcell for calculating the permeability of a porous medium. They considered a lithology factor termed the turtuosity to be a function of saturation, to correct for deviation of the path length from the length of the porous medium. Fatt and Dykstra reported that their equation fit their data as well as the data of Gates and Lietz more accurately than other proposed models.

In 1953, Burdine[6] published equations based on Fatt and Dykstra method for calculating relative permeability from pore size distribution data for both the wetting and nonwetting phases. Burdine related the tortuosity factor to the physical properties of the rock and the saturations of fluids.

In 1958, Wyllie and Gardner[55] generalized Kozeny-Carman equation for the calculations of any wetting and nonwetting relative permeability. They have also suggested an equation for calculating relative permeability to water or oil in water-wet system when one relative permeability is available.

The next section presents a review of the empirical equations that can be used to represent experimental data of water-oil relative permeability. The wetting and non-wetting phases are assumed for water and oil fluids, respectively.

2.1 Wyllie

In 1951, Wyllie[12,54] suggested simple empirical water-oil relative permeability equations for the displacement of oil by water imbibition in consolidated and unconsolidated sands, and in oolitic and vugular lime rocks. Wyllie did not readily check his equations against experimental data because few reliable relative permeability figures only have appeared in the literature then. Wyllie's empirical equations for water and oil relative permeabilities were classified in terms of lithology and pore size distribution and they were given as:

For unconsolidated sand, well-sorted:

$$k_{rw} = (S_w^*)^3 \tag{2.1}$$

$$k_{ro} = (1 - S_w^*)^3 \tag{2.2}$$

For unconsolidated sand, poorly-sorted:

$$k_{rw} = (S_w^*)^{3.5} \tag{2.3}$$

$$k_{ro} = (1 - S_w^*)^2 (1 - S_w^{*1.5}) \tag{2.4}$$

For consolidated sand, oolitic lime, and vugular lime:

$$k_{rw} = (S_w^*)^4 \tag{2.5}$$

$$k_{ro} = (1 - S_w^*)^2 (1 - S_w^{*2}) \quad (2.6)$$

where

$$S_w^* = \frac{S_w - S_{wi}}{1 - S_{wi}} \quad (2.7)$$

2.2 Pirson

In 1958, Pirson[36] derived equations from petrophysical properties of the wetting and non-wetting phase relative permeabilities in clean, water-wet, granular rocks for both drainage and imbibition processes. The relative permeability equations for water-oil imbibition displacement in a water-wet system were given as:

$$k_{rw} = (S_w^*)^{0.5} S^3_w \quad (2.8)$$

$$k_{ro} = (1 - S_w^*) \left[1 - (S_w^*)^{0.25} S_w^{0.5}\right]^2 \quad (2.9)$$

The relative permeabilities for water-oil drainage process were given as:

$$k_{rw} = (S_w^*)^{0.5} S_w^4 \quad (2.10)$$

$$k_{ro} = \left[\frac{1 - S_w - S_{or}}{1 - Sw_i - S_{or}}\right]^2 \qquad (2.11)$$

where S_w^* is as defined in Eq. (2.7).

2.3 Naar et al.

In 1961, Naar et al.[31-33] presented mathematical models, based on the work of Wyllie and Gardener, for predicting two-phase imbibition relative permeability characteristics of consolidated sandstones. Their models were based on the concept of the trapping of the non-wetting phase by the invading wetting phase during the imbibition process. Assuming the trapped non-wetting phase is small to be neglected as shown by Talash[47] for low tension systems, the imbibition water-oil relative permeability equations developed by Naar et al. can be written as:

$$k_{rw} = S_w^{*4} \qquad (2.12)$$

$$k_{ro} = S_o^{*3} (S_o^* + 3 S_w^*) \qquad (2.13)$$

or

$$k_{ro} = S_o^{*3} (3 - 2 S_o^*)$$

where S_w^* is given by Eq. (2.7), and

$$S_o^* = \frac{S_o}{1 - S_{wi}} \tag{2.14}$$

Equation (2.13) is similar to the Corey's[9] equation for the drainage case.

2.4 Jones

In 1966, Jones[17] proposed a mathematical relationship for water-oil relative permeabilities as a function of S_w and S_{wi}, where S_w may be determined from well logs, and S_{wi} may be estimated from an S_{wi} - ϕ crossplot. The correlations proposed by Jones were given as:

$$k_{rw} = (S_w^*)^4 \tag{2.15}$$

$$k_{ro} = \left(\frac{0.9 - S_w}{0.9 - S_{wi}}\right)^2 \tag{2.16}$$

where S_w^* is defined by Eq. (2.7).

2.5 Land

In 1968, Land[25,26] derived equations for two-phase imbibition relative permeability from established theory, by Wyllie et al.[55] and modified by Naar

et al.[31,32], of the dependence of relative permeability on pore-size distribution. The common forms of Land's equations for imbibition water-oil relative permeability in a water-wet system were given as:

$$k_{rw} = (S_w^*)^4 \tag{2.17}$$

$$k_{ro} = S_{o_f}^{*2} \left[1 - (1 - S_{o_f}^*)^2\right] \tag{2.18}$$

where,

$$S_{o_f}^* = \frac{1}{2}\left[(S_o^* - S_{o_r}^*) + \sqrt{(S_o^* - S_{o_r}^*)^2 + \frac{4}{C}(S_o^* - S_{o_r}^*)}\right] \tag{2.19}$$

$$S_{o_r}^* = \frac{S_{o_r}}{1 - S_{w_i}} \tag{2.20}$$

$$C = \frac{1 - S_{w_i} - S_{o_r}}{S_{o_r}} \tag{2.21}$$

S_w^* and S_o^* are defined in Eqs. (2.7) and (2.14), respectively.

2.6 Honarpour et al.

In 1982, Honarpour et al.[16,18] developed a set of empirical prediction equations for water-oil imbibition relative permeability and gas-oil drainage relative permeability using experimentally obtained data. The data used to

arrive at the correlations were derived from oil and gas fields in the United States, Alaska, Canada, Libya, Iran, Argentina, and United Arab Emirates. All the laboratory tests are made at room temperature and atmospheric pressure, on consolidated rocks. Table 2.1 summarizes the relative permeability data used in Honarpour et al. study for water-oil systems. The ranges of rock properties and fluid saturations used in developing the equations for water-oil systems are given in Table 2.2.

Honarpour et al. reported that their equations have not yet been extensively tested. They also reported that their procedure in linear regression analysis was designed to produce a satisfactory fit of the data and it was not intended to provide the best possible data fit. Their study indicated that their correlations better reproduced laboratory relative permeability data than those developed by Wyllie and others in consolidated rocks.

The empirical correlations for relative permeability in water-oil system are presented below. For sandstone and conglomerate rocks, the following correlations were obtained based on wettability of the system.

Water-wet:

$$k_{rw} = 0.035388 \left(\frac{S_w - S_{wi}}{1 - S_{wi} - S_{or}}\right) - 0.010874 \left(\frac{S_w - S_{or}}{1 - S_{wi} - S_{or}}\right)^{2.9}$$

$$+ 0.56556 (S_w)^{3.6} (S_w - S_{wi}) \qquad (2.22)$$

Table 2.1: Summary of Water-Oil Relative Permeability Data Used in Honarpour et al. Equations for Water-Oil Systems

Eq.	k_r predicted	Number of Data Sets	Number of Data Points	Lithology	Wettability
2.22	k_{rw}	84	361	Sandstone & conglomerate	Water
2.23	k_{rw}	101	478	Sandstone & conglomerate	Oil & intermediate
2.24	k_{ro}	185	1000	Sandstone & conglomerate	Any
2.25	k_{rw}	8	57	Limestone & dolomite	Water
2.26	k_{rw}	26	197	Limestone & dolomite	Oil & intermediate
2.27	k_{ro}	54	593	Limestone & dolomite	Any

Table 2.2: Ranges of Rock and Fluid Properties Used in Honarpour et al. Equations for Water-Oil Systems

Eq.	ϕ, %	k_a, md	S_w, %	S_{or}, %
2.22	9.9-30.3	4.10-2640	6.7-70.0	16.4-51.4
2.23	9.1-37.1	0.20-4000	3.6-64.0	7.3-50.0
2.24	9.1-37.1	0.20-4000	3.6-70.0	7.3-56.0
2.25	10.1-15.7	0.05-800	18.5-43.2	10.0-36.4
2.26	8.0-29.1	0.04-490	6.5-40.5	12.7-46.0
2.27	6.5-31.1	0.05-713.4	8.6-43.2	10.0-53.7

Oil-wet and intermediate:

$$k_{rw} = 1.5814 \left(\frac{S_w - S_{wi}}{1 - S_{wi}}\right)^{1.91} - 0.58617 \left(\frac{S_w - S_{or}}{1 - S_{wi} - S_{or}}\right)(S_w - S_{wi})$$

$$- 1.2484\ \phi\,(1 - S_{wi})\ (S_w - S_{wi}) \qquad (2.23)$$

All wetting conditions:

$$k_{ro} = 0.76067 \left[\frac{\frac{S_o}{1-S_{wi}} - S_{or}}{1 - S_{or}}\right]^{1.8} \left(\frac{S_o - S_{or}}{1 - S_{wi} - S_{or}}\right)^{2.0}$$

$$+ 2.6318\ \phi\,(1 - S_{or})\ (S_o - S_{or}) \qquad (2.24)$$

The empirical correlations for predicting water-oil relative permeability curves in limestone and dolomite rocks were also obtained as a function of wettability. They were given as presented below.

Water-wet:

$$k_{rw} = 0.0020525 \left(\frac{S_w - S_{wi}}{\phi^{2.15}}\right) - 0.051371\ (S_w - S_{wi}) \left(\frac{1}{K_a}\right)^{0.43} \qquad (2.25)$$

Oil-wet and intermediate:

$$k_{rw} = 0.29986 \left(\frac{S_w - S_{wi}}{1 - S_{wi}}\right) - 0.32797 \left(\frac{S_w - S_{or}}{1 - S_{wi} - S_{or}}\right)^2 (S_w - S_{wi})$$

$$+ 0.413259 \left(\frac{S_w - S_{wi}}{1 - S_{wi} - S_{or}}\right)^4 \qquad (2.26)$$

All wetting conditions:

$$k_{ro} = 1.2624 \left(\frac{S_o - S_{or}}{1 - S_{or}}\right) \left(\frac{S_o - S_{or}}{1 - S_{wi} - S_{or}}\right)^2 \qquad (2.27)$$

2.7 Chierici Models

In 1984, Chierici[7,8] proposed empirical models for estimating water-oil imbibition relative permeability curves. The proposed models were shown to match the experimentally determined curves, in particular at and near their initial points and endpoints, better than Land's models.

$$k_{ro}^{*} = \text{Exp}\,[\,-A\ R_w^{L}\,] \qquad (2.28)$$

$$k_{rw}^{*} = \text{Exp}\,[\,-B\ R_w^{-M}\,] \qquad (2.29)$$

where,

$$R_W = \frac{S_w - S_{wi}}{1 - S_{or} - S_w} \tag{2.30}$$

$$k_{ro}^{*} = \frac{k_{ro}}{k_{ro(Swi)}} \tag{2.31}$$

$$k_{rw}^{*} = \frac{k_{rw}}{k_{rw(Sor)}} \tag{2.32}$$

The values of the empirical coefficients A, L, and B, M are to be determined by nonlinear regression on the sets of experimental data points. Where $k_{ro}(S_{wi})$ is the relative permeability to oil at $S_o = 1 - S_{wi}$, and $k_{rw}(S_{or})$ is the relative permeability to water at $S_w = 1 - S_{or}$.

Kumar et al.[23] used the above models to reproduce relative permeability data for high- and low-tension systems at elevated temperature. They reported that data of relative permeability estimated by these models agreed well with the limited experimental data used in their study.

CHAPTER 3

CHAPTER 3

AN OVERVIEW OF TWO-PHASE RELATIVE PERMEABILITY

3.1 Methods of Measurements

The relative permeability of a rock to each fluid phase can be measured in a core sample in the laboratory by either "steady-state" or "unsteady-state" methods. Representative cores should be obtained from each stratum of a reservoir rock to be used in laboratory measurements. Native-state and restored state cores are common methods for measuring the relative permeabilities. The following discusses these methods in details. [1,2,10,15,30,34,53]

3.1.1 Native-state and Restored-state Cores

In the native-state core[10], the rock sample retains the wettability and fluid saturation existing at the well-site. This might not be the actual wettability

of the formation. In the restored state, the core sample is cleaned and dried, and the fluid saturations restored to conditions thought to exist in the reservoir. The wettability of the sample might or might not be like reservoir conditions' wettability.

In the laboratory, the native-state cores are cleaned dynamically with solvent and alcohol to remove both oil and water. The alcohol is followed by formation water, which in turn is displaced by viscous oil to establish a connate water saturation. This method is believed to provide wettability and saturation conditions close to that of the initial reservoir conditions if the rock wettability has not been altered during coring.

Core samples for restored-state tests are cleaned and dried before saturating with formation water. The connate water saturation can be established with the same method used for native-state cores, or by a gas evaporation technique. The water saturation established using the gas evaporation technique is based on air permeability-water saturation relationships from capillary pressure tests. The restored-state tests can change clay minerals and alter the wettability of the sample.

Native-state cores are preferred to provide a close representation of reservoir wettability, which is crucial for obtaining realistic permeability data. Fresh-state samples may also have wetting characteristics similar to those in the reservoir provided that an appropriate coring fluid is used. However, flushing by mud filtrate generally changes the initial water saturation.

Coring operations should be designed to minimize mud filtration so that undesirable flushing before laboratory testing is avoided. Using large-diameter cores also reduces the influence of flushing by drilling muds and minimizes the

core contamination. Also, the recovered cores should be preserved without unnecessary delays to avoid weathering that may result in wettability changes.[22,19]

Enough core samples should be selected to cover the entire range of rock properties evident in the formation. If longer samples are not available, a composite core can be made by placing several closely matched plugs in series to perform the experimental relative permeability measurements.

3.1.2 Steady-state Method

The measurement of relative permeability basically involves determining the flow rate of oil, water, or gas at a known fluid saturation and a specified pressure differential. From these measurements, the fluid relative permeabilities can be calculated.

Early experimenters suggested techniques that would fix the fluid saturation and measure the corresponding relative permeabilities. These techniques included the Hassler method, the Penn State method, the stationary liquid procedure, the single core dynamic, the intermittent gas drive, the Hafford method, and the dispersed feed procedure. Many of these methods are closely related and attempt to eliminate both inlet and outlet (boundary) end effects, which appear as saturation gradients. It has been reported that boundary effects could be eliminated and that the Penn State, Hafford, Hassler, and dispersed feed techniques can give valid results. It was further observed that drainage relative-permeability characteristics were independent of flow rate

as long as the rate was not so high as to cause inertial effects. Other experiments confirmed that laboratory-measured relative permeabilities are independent of flow rate, provided that there is no saturation gradient induced in the core sample by boundary effects. Furthermore, relative-permeability characteristics were found to be independent of fluid viscosities. However, since relative permeabilities depend on the direction of saturation change, the direction of change in the laboratory testing must correspond to that in the reservoir.[10]

In the steady-state method[3], water and oil are injected simultaneously at constant rates or pressure with different proportionality till equilibrium is reached. The saturations, flow rates, and pressure gradients are measured and Darcy's equation is used to determine the effective permeability for each phase (Fig. 3.1a). Relative permeability equations for both oil and water are as follows:

$$k_{ro} = \frac{ko}{ko(swi)} = \frac{q_o \mu_o L}{A \Delta P} \qquad (3.1)$$

$$k_{rw} = \frac{kw}{ko(swi)} = \frac{q_w \mu_w L}{A \Delta P} \qquad (3.2)$$

where,

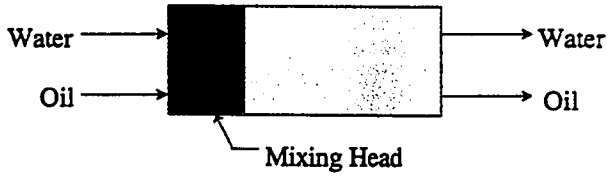
k_{ro} = relative permeability to oil, fraction.

k_{rw} = relative permeability to water, fraction.

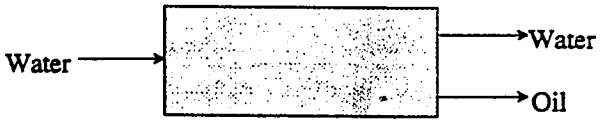
k_o = effective permeability to oil, md.

k_w = effective permeability to brine, md.

$k_{o(swi)}$ = effective permeability to oil at irreducible water saturation (base permeability), md.



(a)- Steady-State Method



(b)- Unsteady-State Method

Figure 3.1- Description of Steady-State and Unsteady-State Methods

q_o = oil flow rate through core, cc/min.

q_w = water flow rate through core, cc/min.

μ_o = oil viscosity, cp.

μ_w = water viscosity, cp.

ΔP = measured pressure drop across the core, psi.

A = cross-sectional area of the core, cm^2.

L = length of the core, cm.

Other conversion constants to be included in the Darcy's equations have been factored out.

The steady-state method has the greatest reliability and the ability to determine relative permeability for a wider range of saturation levels because the capillary pressure prevails, the saturation is measured directly, and the calculation scheme is based on Darcy's law. However, the steady-state method is inherently time-consuming because achieving equilibrium may require several hours or days at each saturation level. In addition, these methods require independent measurements of fluid saturations in the core.[19]

3.1.3 Unsteady-state Methods

In 1952, Welge[52] presented a new technique for determining the relative permeability characteristics of a rock sample from its performance during an external gas or water drive. Using the Buckley-Leverett[5] frontal advance equations, Welge developed relationships between the average saturation, the

producing-end saturation, and the amount of fluid injected into a core sample. From this experimental data, the producing water-oil ratio (WOR) or the gas-oil ratio (GOR) can be calculated and the corresponding relative permeability ratio can then be determined. This relative permeability ratio can be then be related to the corresponding outflow face saturation.

The merit of the unsteady-state techniques is its simplicity and speed. In this technique, the saturation equilibrium is not required to be attained and, therefore, the entire set of relative-permeability curves can be obtained in a few hours. Usually, the process involves displacing in-situ fluids by constant-rate or constant-pressure injection of a driving fluid while monitoring the effluent volumes continuously (Fig. 3.1b). The production data are analyzed, and a set of relative-permeability curves is determined using various mathematical methods. The Welge[52], Johnson-Bossler-Naumann[20], and Jones-Raszelle[21] methods are most commonly used for analysis. The main advantages of unsteady-state techniques include fewer instrumentation requirements and substantially reduced test times compared with steady-state tests.

However, unsteady-state techniques have operational problems such as capillary end effects, viscous fingering, and channeling in heterogeneous cores which are difficult to monitor and to account for properly. Unless the mobility (the ratio of effective permeability to viscosity) of the displacing fluid is much higher than that of the in-situ fluids, the time between the front breakthrough and complete floodout is usually small, which introduces computational difficulties.[19]

In designing experiments to determine relative permeability by the unsteady-state method, it is necessary that the pressure gradient be large enough

to minimize capillary pressure end effects, the pressure differential across the core be sufficiently small compared with total operating pressure so that compressibility effects are insignificant, and the core be homogeneous.

3.2 Factors Affecting Measurements of Relative Permeability

Careful design of the apparatus and operating conditions in the laboratory is essential for accurate relative permeability measurements. Special attention should be given to problems such as capillary end or boundary effects, hysteresis, and scaling effects.

3.2.1 Capillary End Effect[19,35,53]

The most important factor influencing the relative permeability measurements is the capillary end effect, or sometimes called boundary effect. The end effect is caused by the discontinuity in the pressures of the wetting and nonwetting phases at the end of the core. In otherwords, it occurs from being the saturation of the wetting phase higher close to the inlet and outlet ends of the rock samples. These higher saturations at the ends are the result of greater affinity of the wetting phase to remain in pore capillaries rather than to exit to a noncapillary space. Where a saturation gradient exists in a core sample, the actual pressure drop across the core is less than that causing flow of the

nonwetting phase by an amount equal to the difference in capillary pressure at the two ends of the core. Neglecting this fact and assuming the pressure drop in the wetting phase to be that in the nonwetting phase, the wetting relative permeability computed by use of this assumption is too low. Another influence of the boundary effect results from the existence of a saturation gradient. Where such a gradient exists, the relative permeabilities to both phases flowing vary throughout the core with changes in the saturation. If these variations be disregarded, the relative permeabilities to both phases at the average saturation are too low.

Several techniques have been proposed to reduce or to minimize end effects. The most obvious way of avoiding or reducing the end effects is to perform all measurements on a section of the core sample far enough removed from the outflow end to ensure that the saturation gradient in the test section is insignificant. In practice, this may be achieved by use of a single core or placing beyond the test core a core sample of similar material.

Other techniques for reducing or minimizing the influence of end effects include displacement at high flow rates so that the influences of viscous forces become much greater than capillarity, and use of longer cores while restricting the pressure and saturation measurements to the inner sections of the cores.

Another method of avoiding the boundary effect is to hold the wetting phase stationary by capillary forces and flowing only the non-wetting phase.

Still another technique of avoiding or minimizing the boundary effect is to maintain a uniform saturation distribution throughout the core by controlling the capillary pressure at both ends of the core sample. This is accomplished by placing a porous material at both ends of the test core.

Rule-of-thumb criteria based on laboratory experience are also used to select operating conditions that minimize capillary end effects. The end effect is considered insignificant when flow is stabilized. Rapoport and Leas[41] developed scaling limits' criteria based on dimensional analysis to examine the stabilized flow from displacement data. They showed that the stabilized flow was achieved when the numerical value of the scaling factor, $L\mu v$, exceeded a critical value. Critical values of scaling factors obtained in laboratory experiments range from 0.4 to 5.0 cm^2cp/min.

Other criteria for selecting the optimum operating conditions that minimize capillary end effects are those proposed by Kyte and Rapoport[24]. They recommended that the displacement tests be performed at an overall pressure drop of 50 psi or greater, or the frontal velocities (v/ϕ) be maintained around 0.635 to 1.06 cm/min.

3.2.2 Hysteresis[4,10,19,53]

Relative permeability-saturation relations are not unique functions of saturation for a given core, but are subject to hysteresis; that is, the dependence of relative permeability values on saturation history. Errors caused by hysteresis may seem easy to eliminate, but they are practically difficult to control. Since hysteresis does exist, careful design of the experimental procedure must be exercised that the relative permeabilities measured in the

laboratory are obtained under conditions where each saturation is approached in the desired manner.

The type of relative permeability curve that is representative of flow characteristics through the formation in the reservoir depends on the mechanism by which the reservoir is depleted. If the reservoir is depleted by decreasing the oil saturation and increasing the water saturation, as in a water-drive mechanism, the imbibition relative permeability curves should be used. If, however, the reservoir is depleted by decreasing the oil saturation and increasing gas saturation, as in a dissolved-gas drive or expanding gas-cap drive, the drainage relative permeability curves should be applied.

Relative permeability curves for drainage and imbibition paths are shown in Figures 3.2 and 3.3, respectively. Arrows indicate the direction of saturation change. The curves labeled NW and W corresponds to the nonwetting phase and wetting phase, respectively. The base permeability is the permeability to the nonwetting phase (oil) at interstitial wetting-phase (water) saturation. The paths followed to generate the relative permeability data are 1 to 2 and 2 to 3 as noted in Figures 3.2 and 3.3. The relative permeability of the wetting phase may be independent of the saturation path. That is, during imbibition, the wetting phase permeabilities retrace those obtained during drainage to the maximum wetting phase saturation. This phenomenon is usually observed in systems with strong wettability preference. However, the relative permeability of the nonwetting phase is quite dependent on the saturation path over the same saturation range. That is, the nonwetting phase has a lower relative permeability at any saturation during imbibition than it does during drainage.

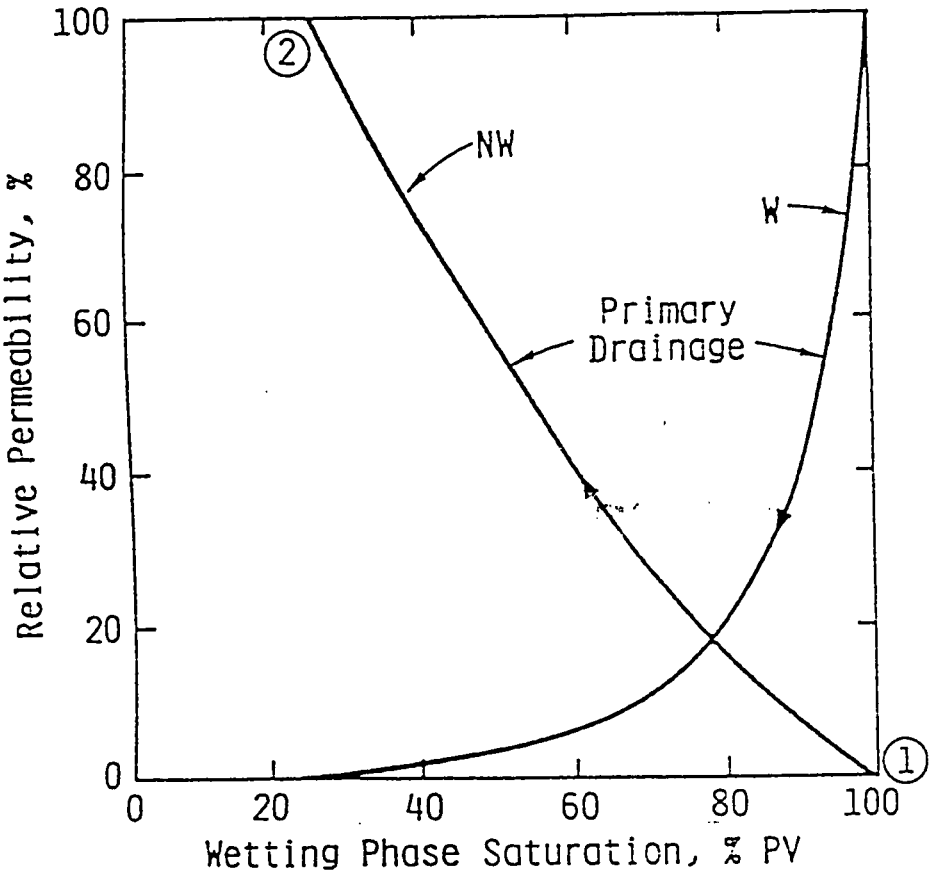


Fig. 3.2: Primary Drainage Relative Permeability Curves for a Strongly Water-Wet Rock (After Wilihite).

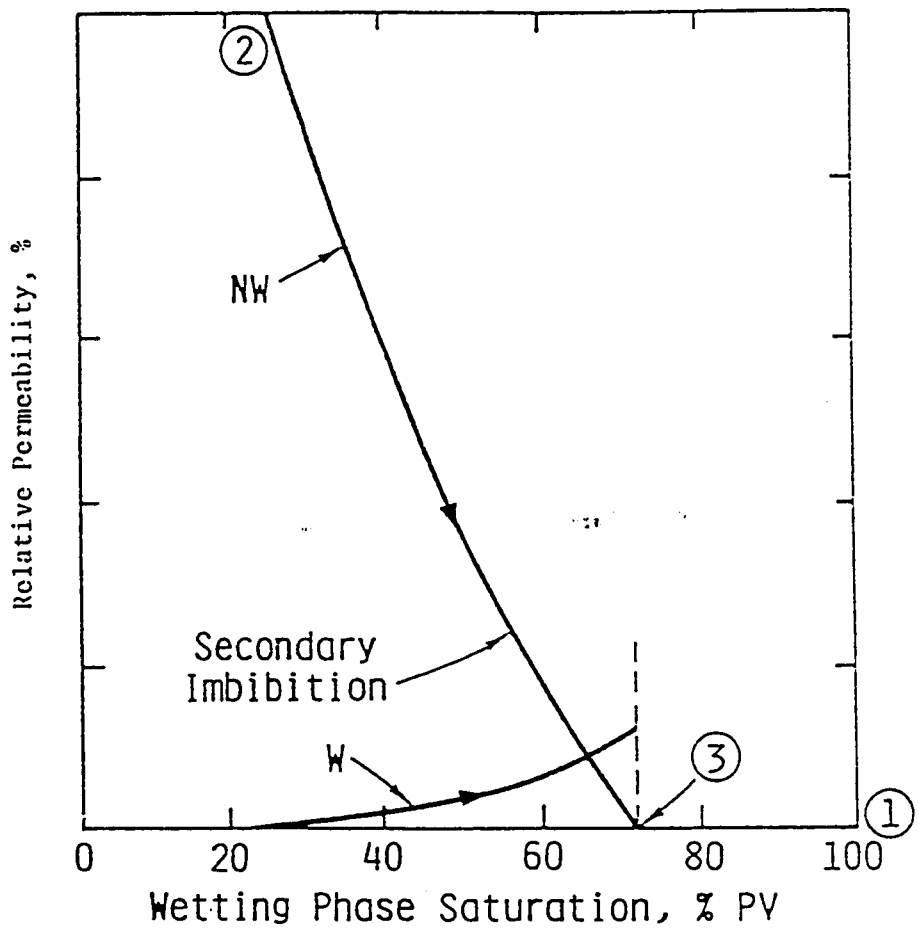


Fig. 3.3: Secondary-Imbibition Relative Permeability Curves for a Strongly Water-Wet Rock (After Willhite).

3.2.3 Wettability

Wettability is a measurement of the ability of a fluid to coat the rock surface. It is the main factor responsible for the microscopic fluid distribution in porous media; thus it has a significant effect on displacement behavior and relative permeability characteristics. The effect of wettability on relative permeability has been investigated by several authors[35,50,17,53]. Owens and Archer[35] showed, Figure 3.4, that an increase in oil wetness resulted in an increase in water relative permeability and a decrease in oil relative permeability at a given water saturation.

Craig[10] provided the following rules-of-thumb criteria for indicating the wettability preferences in a water-oil system:

	Water-wet	Oil-wet
S_{wc}	> 20-25%	< 15%, usually < 10%
S_w at $k_{rw}=k_{ro}$	> 50 %	< 50 %
k_{rw} at S_{or}	< 0.3	> 0.5, approaching 1.0

Intermediate wettability systems have mixed characteristics of both water-wet and oil-wet systems.

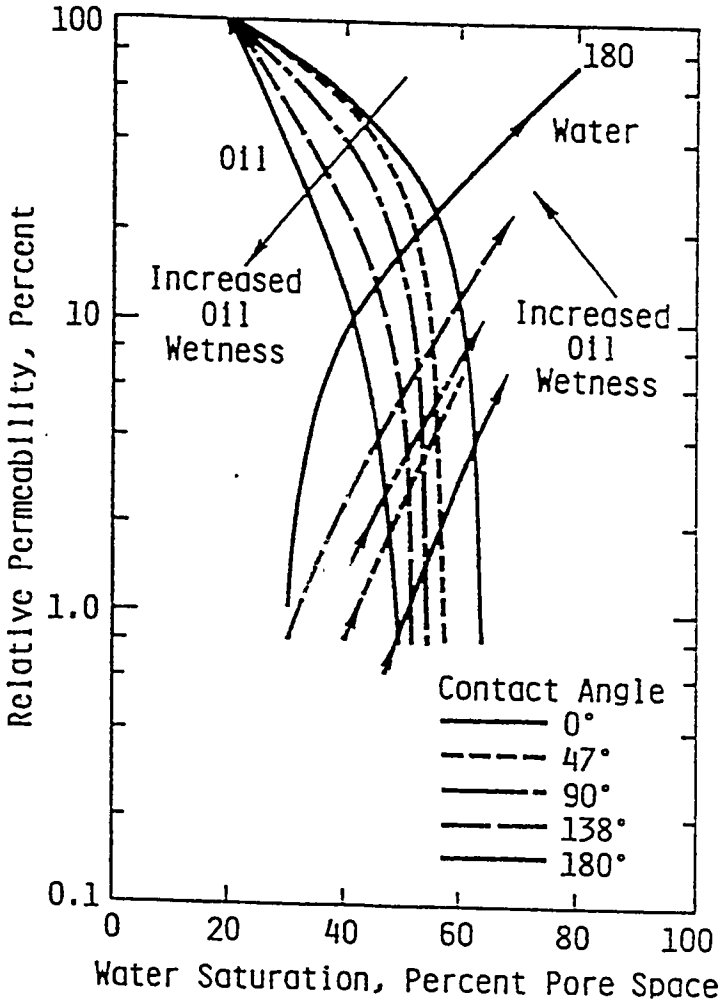


Fig. 3.4: The Effect of Wettability on Water-Oil Relative Permeability Curves (After Archer and Owens).

3.2.4 Temperature

The effect of temperature on relative permeabilities has been studied by several investigators. Sufi et al.[46] studied the effect of temperature on relative permeability on unconsolidated sands. They concluded that the relative permeability was unaffected by temperature changes between 70 and 186 °F.

Lo and Mungan[28] conducted steady-state relative permeability measurements on consolidated Teflon and Berea sandstone. Their results indicated that the relative permeabilities were dependent of temperature when using white oils, but were independent of temperature when using tetradecane.

Poston et al.[37] performed a series of unsteady-state displacement tests on unconsolidated sand at elevated temperature. The results showed that the relative permeabilities to both oil and water generally increased with increasing temperatures.

3.2.5 Viscosity and Viscosity Ratio

Many studies investigated the effect of viscosity and viscosity ratio on relative permeability curves. The results generally concluded that the relative permeability curves are independent of the viscosity ratio for the displacement of the nonwetting phase by the wetting phase, and dependent of the viscosity ratio for the displacement of wetting phase by a nonwetting phase.

Leverett[27] studied the effect of viscosity ratio on the oil and water behavior from measurements of steady-state displacement tests in

unconsolidated sandpacks. He found that the relative permeabilities were independent of oil to water viscosity ratios varying from 0.057 to 90.0, as shown in Figure 3.5. Richardson[42] observed from steady-state displacement experiments that relative permeability ratios were not influenced by oil viscosity values ranging from 1.8 to 151 cp as depicted in Figure 3.6.

In his study, Leverett performed a series of displacement experiments using Teflon cores at viscosity ratios of 1, 12, and 20. The results indicated that the viscosity ratio had substantial effects on relative permeability ratios when the wetting phase was displaced by the nonwetting phase.

3.2.6 Scaling

Relative permeability measurements should be performed under representative reservoir conditions to make use of the laboratory data in scaling up to the field level. This involves performing the tests with the appropriate combinations of viscous, capillary, and gravity forces such that a stable displacement through the core is warranted, while at the same time the similarity in the microscopic flow behavior between the reservoir and the core is still maintained. This objective can be achieved by applying linear scaling criteria. Measurements of relative permeability conducted at room temperature using dead crude or even refined oil can sometimes also be useful. In such cases, it is necessary that sufficient tests under simulated reservoir conditions are performed to evaluate the reliability of such idealized tests.[19]

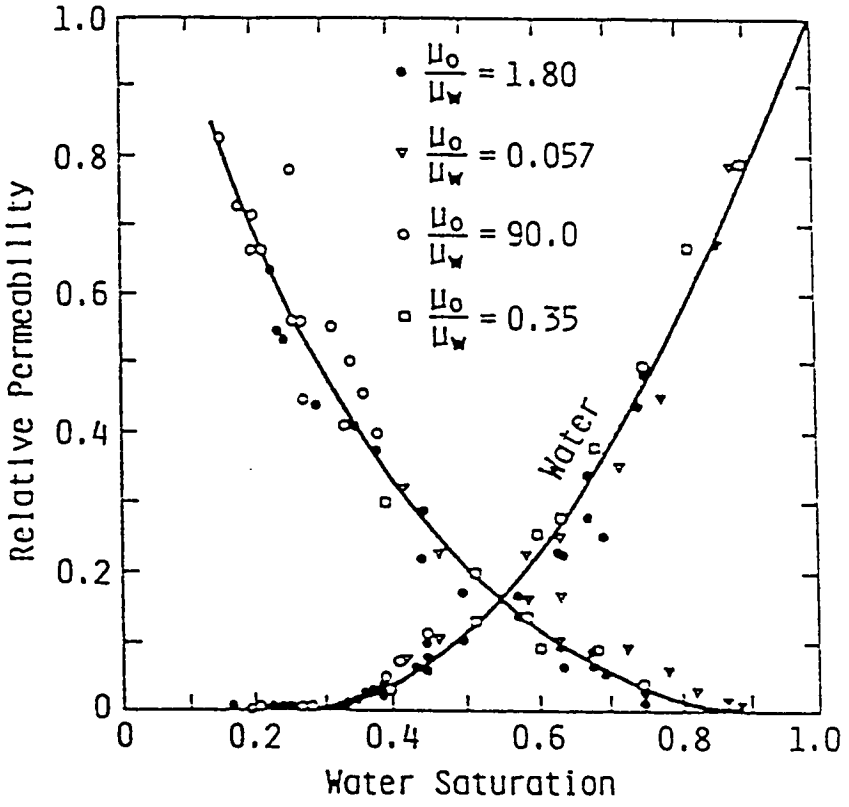


Fig. 3.5: The Effect of Viscosity Ratio on Water-Oil Relative Permeabilities (After Amyx et al.).

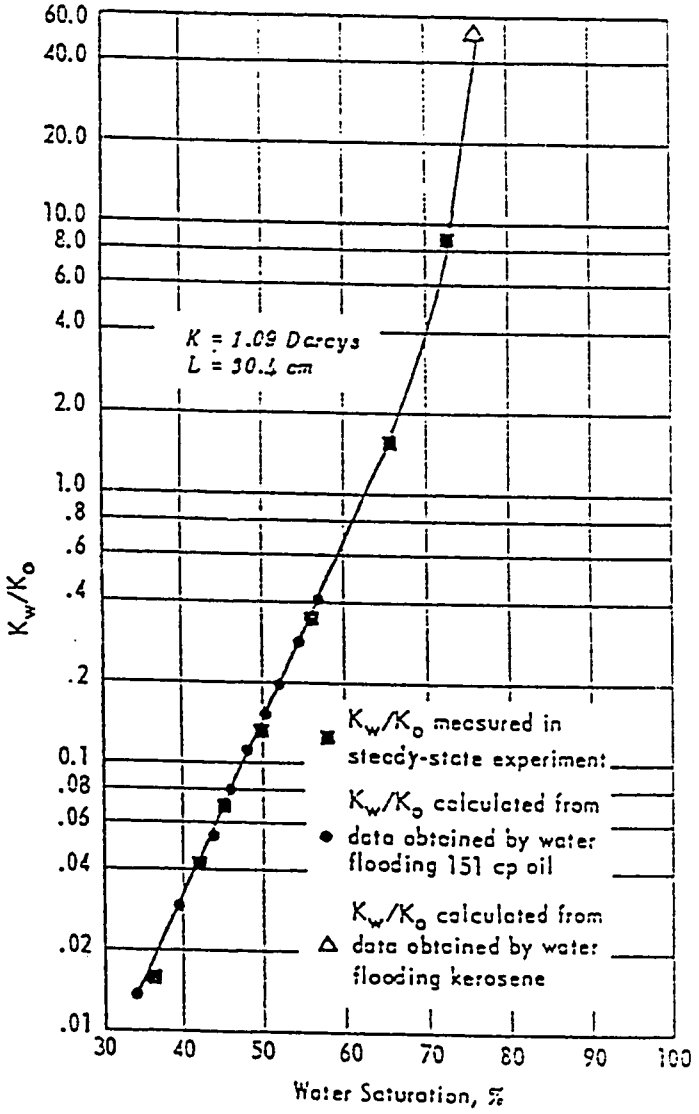


Fig. 3.6: The Effect of Oil Viscosity on Relative Permeability Ratio of Water to Oil (After Leverett).

3.3 Determination of Relative Permeability Curves

An important application of the frontal advance solution is the calculation of relative permeability ratios and individual relative permeability curves from linear displacement data obtained in the laboratory using core samples. The data must be obtained under conditions where stabilized flow is attained and capillary end effects are minimized.

Individual relative permeabilities may be calculated from linear displacement data when the pressure differences across the core are measured during displacement at constant-rate or constant-pressure displacement experiments (unsteady-state method). Johnson-Bossler-Naumann[20] "JBN" developed the first method of determining oil and water relative permeabilities from the analysis of waterflood performance. Jones and Raszelle[21] developed a graphical technique that is equivalent to the equations of Johnson et al., but the base permeability is the permeability to oil at connate water saturation in JBN method. The following describes the JBN technique for determining oil and water relative permeabilities from the analysis of displacement measured data.

Because the JBN method is based on the Buckley-Leverett solution and the work of Welge, relative permeabilities can be obtained for any saturation greater than or equal to the breakthrough water saturations, S_{wf}. In this method, the fractional flow of oil at the outlet face of the core is given by:

$$f_o = \frac{d\bar{S}_w}{dQ_i} = \frac{1}{1 + \frac{\mu_o k_{rw}}{\mu_w k_{ro}}} \qquad (3.3)$$

where,

$\bar{S}_w$ = average water saturation, fraction

Q_i = cumulative water injected, PV

k_{ro} = oil relative permeability

k_{rw} = water relative permeability

μ_o = oil viscosity, cp

μ_w = water viscosity, cp

The water saturation at the outlet of the core (S_w) is obtained from:

$$S_w = \bar{S}_w - Q_i f_o \tag{3.4}$$

The average water saturation, $\bar{S}_w$, can be found from a material balance for water content in the porous media:

$$\bar{S}_w = S_{wi} + Q_o \tag{3.5}$$

where,

S_{wi} = initial water saturation, fraction

Q_o = cumulative oil produced, PV

Individual phase permeabilities corresponding to S_{w2} are computed from the overall pressure-drop data that are measured from the beginning of the

waterflood as a function of Q_i. Johnson et al. defined the relative injectivity of the displacing fluid as:

$$I_r = \frac{\frac{q}{A\Delta P}}{\frac{q_s}{A\Delta P_s}} = \frac{\text{injectivity}}{\text{initial injectivity}} \qquad (3.6)$$

where,

I_r = relative injectivity

q = volumetric displacement rate, cm^3/hour

q_s = volumetric displacement rate at start of waterflood, cm^3/hour

ΔP = overall pressure drop, psi

ΔP_s = overall pressure drop at start of waterflood, psi

Johnson et al. show that the oil-phase relative permeability is related to the process variables by:

$$\frac{f_o}{k_{ro}} = \frac{d\left(\frac{1}{Q_i I_r}\right)}{d\left(\frac{1}{Q_i}\right)} \qquad (3.7)$$

Values of k_{ro} and S_{w2} can be determined from the experimental data by differentiating the graph of $(1/Q_i I_r)$ vs. $(1/Q_i)$ graphically or numerically. The relative permeability of the water phase is computed from:

$$k_{rw} = \left(\frac{1 - f_o}{f_o}\right) \frac{\mu_w}{\mu_o} k_{ro} \qquad (3.8)$$

Sometimes there are discontinuities in the experimental data. In such cases, it is necessary to smooth the data before differentiation. Data may be smoothed by hand with the appropriate curve or preferably with numerical techniques. Tao et al.[48] used a Monte Carlo error analysis to study the effect of different experimental operating conditions on the accuracy of relative permeabilities computed by the JBN technique. They[49] provided algorithms that can be used for computer implementation and calculation of derivatives of measured data in the JBN technique.

A computer program for determining the water-oil relative permeabilities using the above procedure of the JBN method is developed in this study and tested with actual experimental data.

An example calculation is provided in the Appendix to illustrate the use of JBN method to obtain water-oil relative permeability curves for a constant-rate displacement. Experimental data published in the literature[53] were used for this purpose.

CHAPTER 4

CHAPTER 4

THEORY OF REGRESSION MODELS AND ANALYSIS

Regression analysis is one technique of modeling the relationship between one variable and another set of variables. The relationship is expressed as an equation that predicts the response variable "the dependent variable" from a function of regressor variables "the independent variables" and parameters. Empirical equation or correlation analysis measures the strength of such relationship by means of statistical analysis.[29,38,44,51]

4.1 Linear Multiple Regression[51]

A multiple linear regression model is a model with more than one independent variable and is linear in the coefficients. The main objectives to be accomplished in using multiple linear regression are to:

1. screen variables to determine which have a significant effect on the response,
2. obtain estimates of individual coefficients in a complete model, and
3. arrive at the most effective prediction equation.

In the first objective the prediction of response or experimental measurements is secondary, while in the third individual regression coefficients are not as important as the quality of the estimated response $\hat{y}$. In the second target it is known a priori that all variables are to be included in the model.

For k independent variables, $x_1, x_2, ..., x_k$, the mean of $y(x_1, x_2, ..., x_k)$ is given by the multiple linear regression model:

$$\mu_Y(x_1, x_2, ..., x_k) = \beta_0 + \beta_1 x_1 + ... + \beta_k x_k \qquad (4.1)$$

and the estimated response, $\hat{y}$, is obtained from the sample regression equation:

$$\hat{y} = b_o + b_1 x_1 + ... + b_k x_k \qquad (4.2)$$

where each regression coefficient β_i is estimated by b_i from the sample data using the method of least squares. The n equations for the n experimental measurements can be expressed in matrix form as:

$$y = X\beta \qquad (4.3)$$

or,

$$\begin{bmatrix} y_1 \\ y_2 \\ . \\ . \\ . \\ y_n \end{bmatrix} = \begin{bmatrix} 1 & x_{11} & x_{21} & . & . & . & x_{k1} \\ 1 & x_{12} & x_{22} & . & . & . & x_{k2} \\ . & . & . & & & & . \\ . & . & . & & & & . \\ . & . & . & & & & . \\ 1 & x_{1n} & x_{2n} & . & . & . & x_{kn} \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \\ . \\ . \\ . \\ \beta_k \end{bmatrix} \qquad (4.4)$$

Similar least-squares techniques can also be applied in estimating the coefficients when the linear model involves, for example, powers and products of the independent variables.

The least squares estimators of the parameters β_o , β_1 , ... , β_k are obtained by fitting the multiple linear regression model, Eq. (4.1), to the data points, $\{ (x_{1i}, x_{2i} , x_{ki} , y_i) ; i = 1, 2, ... , n$ and $n > k \}$. y_i is the observed response to the values x_{1i}, x_{2i} ,... , x_{ki} of the k independent variables x_1, x_2 , x_k. In using the concept of least squares to estimate b_o, b_1 , ... b_k , the error sum of square should be minimized.

$$SSE = \sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} (y_i - b_o - b_i\ x_{1i} - b_2\ x_{2i} ... -\ b_K\ x_{ki})^2 \qquad (4.5)$$

Where e_i represents the residual associated with response y_i. A set of $k + 1$ normal equations are generated by differentiating SSE with respect to b_o, b_1, b_2,

... , b_k , by any appropriate method for solving system of linear equations. Using matrix notation, these equations can be written as:

$$\mathbf{(X'X)b = X'y} \tag{4.6}$$

or, equivalently

$$\begin{bmatrix} n & \sum_{i=1}^{n} x_{1i} & \sum_{i=1}^{n} x_{2i} & \cdots & \sum_{i=1}^{n} x_{ki} \\ \sum_{i=1}^{n} x_{1i} & \sum_{i=1}^{n} x^2_{1i} & \sum_{i=1}^{n} x_{1i}x_{2i} & \cdots & \sum_{i=1}^{n} x_{1i}x_{ki} \\ \vdots & \vdots & \vdots & & \vdots \\ \sum_{i=1}^{n} x_{1i} & \sum_{i=1}^{n} x_{ki}x_{1i} & \sum_{i=1}^{n} x_{ki}x_{2i} & \cdots & \sum_{i=1}^{n} x^2_{ki} \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ \vdots \\ b_k \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n} y_i \\ \sum_{i=1}^{n} x_{1i}y_i \\ \vdots \\ \sum_{i=1}^{n} x_{ki}y_i \end{bmatrix} \tag{4.7}$$

In many regression cases, individual coefficients are of importance to the experimenter. However, the interest is likely not to be in the individual parameter but rather in the ability of the entire function to predict the true response in the range of the variables considered.

One criterion that is commonly used to illustrate the adequacy of a fitted regression model is the coefficient of multiple determination.

It merely indicates the proportionality of the total variation in the experimental data that is explained by the fitted model. The square root of r^2 is called the multiple correlation coefficient, r, between the response and a set of independent variables.

The addition of any single variable to a regression system will increase the regression sum of squares and thus reduce the error sum of squares. Consequently, one must decide whether the increase in regression is sufficient to warrant using it in the model. As one might expect. the use of unimportant variables can reduce the effectiveness of the prediction equation by increasing the variance of the estimated response. The orthogonality of variables and t-tests are significant tests in determining which variables should be used in the final regression model.

4.2 Model Selection Methods [44,51]

There are a number of model selection methods implemented in many statistical computer packages such as SAS. One standard procedure for searching for the optimum subset of variables is a technique called stepwise regression. It is based on the procedure of sequentially introducing the variables into the model one at a time. The description of this procedure will be better understood if the methods of forward selection and backward elimination are described first.

Forward selection starts with no variables in the model and adds variables one by one to the model. At each step, the variable added is the one that maximizes the fit of the model. Thus, variables are added one by one to the model until no remaining variable produces a significant F-statistic, or equivalently t-tests.

Backward elimination involves the same concepts as forward selection except that it starts with all the variables in the model and eliminates variables one by one from the model. At each step, the variable with the smallest contribution to the model is deleted.

Stepwise regression is accomplished with a slight but important modification of the forward selection procedure. The modification involves further testing at each stage to ensure the continued effectiveness of variables that had been inserted into the model at an earlier stage. This represents an improvement over forward selection, since it is quite possible that a variable entering the regression equation at an early stage might have been rendered unimportant or redundant because of relationships that exist between it and other variables entering at later stages. Therefore, at a stage in which a new variable has been added to the regression equation through a significant increase in r^2 as determined by the F-test, all the variables already in the model are subjected to F-tests (or, equivalently to t-tests) in light of this new variable, and are deleted if they do not display a significant F-value. The procedure is continued until a stage is reached in which no additional variables can be inserted or deleted.

Other model selection techniques include the maximum r^2 improvement, minimum r^2 improvement, and r^2 selection.

4.3 Nonlinear Multiple Regression [38,44]

The nonlinear regression is a procedure that produces least-squares or weighted least-squares estimates of the parameters of a nonlinear model. The NLIN procedure available in the Statistical Analysis System[44] "SAS" is used in this study for the development of equations.

This procedure fits nonlinear regression models by least squares. Nonlinear models are usually more difficult to specify and estimate than linear models. The nonlinear procedure requires the model expression, parameter names, initial guess values for parameters, and possibly derivatives of the model with respect to the parameters. The NLIN procedure first examines the initial value specifications of the parameters. If a grid of values is specified, NLIN evaluates the residual sum of squares at each combination of values to determine the best set of values to start the iterative algorithm. Then NLIN implements one of the iterative methods: steepest-descent, Newton, modified Gauss-Newton, Marquardt, or multivariate secant method. The Marquardt method is a reliable technique and works very well in practice and has become the standard of nonlinear least-squares routines. For this, it was the choice among other methods for use in this study.

The Marquardt method is a compromise between Gauss-Newton method and steepest descent method. It regresses the residuals onto the partial derivatives of the model with respect to the parameters until the estimates converge. It is equivalent to performing a series of ridge regressions and is useful when the parameter estimates are highly correlated or the objective function is not well approximated by a quadratic.

Although all the relative permeability models being dealt with in this study are nonlinear, they can be reduced to linear forms by applying the appropriate transformation of variables. Consider the following nonlinear equation, as an example:

$$Y = b_0 \cdot X_1^{b_1} \cdot X_2^{b_2} \cdot e^{b_3 X_3} \tag{4.8}$$

Applying the logarithmic transformation for both sides, Eq.(4.8) can be written as:

$$\log Y = \log b_0' + b_1 \log X_1 + b_2 \log X_2 + b_3 X_3 \tag{4.9}$$

which reduces to a linear equation that can be solved by the linear multiple regression method, as discussed earlier.

$$y = b_0 + b_1 x_1 + b_2 x_2 + b_3 x_3 \tag{4.10}$$

where,

$y = \log(Y)$

$b_0 = \log(b_0')$

$x_1 = \log(X_1)$

$x_2 = \log(X_2)$

$x_3 = X_3$

4.4 Error Analysis [38,44,51]

Statistical and graphical error analyses are used in this study to evaluate the performance and the accuracy of empirical equations developed in this study and by other authors.

4.4.1 Statistical Error Analysis

Various statistical parameters can be used to determine the accuracy of empirical equations. The statistical parameters used in this study were average error, average absolute error, minimum/maximum absolute error, standard deviation, root mean square, coefficient of determination, and the correlation coefficient.

4.4.1.1 Average Error

This is a measure of deviation from the experimental data and is defined by:

$$E_{av} = \frac{1}{n} \sum_{i=1}^{n} E_i \qquad (4.11)$$

E_i is the error, residual, in the fit due to regression and is defined by

$$E_i = (x_{exp} - x_{est})_i \quad , \quad i = 1, 2, \ldots n \tag{4.12}$$

where x_{exp} and x_{est} represent the experimental and estimated values, respectively. The lower the value of E_{av}, the more uniformly distributed are the errors between positive and negative values.

4.4.1.2 Average Absolute Error

This parameter measures the absolute deviation from the experimental data and is defined by

$$E_{ab} = \frac{1}{n} \sum_{i=1}^{n} |E_i| \tag{4.13}$$

A lower value of E_{ab} implies a better correlation.

4.4.1.3 Minimum and Maximum Absolute Error

The minimum and maximum values are determined by scanning the computed absolute error values. These parameters are used to give the range of error for each correlation.

$$E_{min} = \frac{1}{n} \min_{i=1}^{n} |E_i| \tag{4.14}$$

and

$$E_{max} = \frac{1}{n} \max_{i=1}^{n} |E_i| \tag{4.15}$$

A lower value of maximum absolute error indicates a higher accuracy of the correlation.

4.4.1.4 Standard Deviation

Standard deviation is a measure of data dispersion around zero deviation and is given by

$$s^2 = \frac{1}{n-1} \sum_{i=1}^{n} E_i^2 \tag{4.16}$$

A lower value of standard deviation indicates a smaller degree of scatter.

4.4.1.5 Root Mean Square

It is defined by

$$rms = \left[\frac{1}{n} \sum_{i=1}^{n} E_i^2 \right]^{1/2} \tag{4.17}$$

A lower value of root mean square means a narrower variation of data around zero deviation.

4.4.1.6 Coefficient of Determination

The coefficient of determination indicates the proportionality of the total variation in the experimental data that is predicted from or explained by the fitted model. It is defined as

$$r^2 = \sum_{i=1}^{n} (x_{est} - \bar{x})_i^2 \; / \; \sum_{i=1}^{n} (x_{exp} - \bar{x})_i^2 \qquad (4.18)$$

where,

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} (x_{exp})i \qquad (4.19)$$

The square root of r^2 is called the correlation coefficient, r, and it has values between 0 and 1. A value of 1 indicates a perfect correlation whereas a value of 0 implies no correlation at all among the given independent variables. The larger the value of r^2 or r, the greater is the reduction in the sum of squares of errors, and the stronger is the relationship between the independent and dependent variables.

4.4.1.7 F-Test

The F-test tests the hypothesis that two samples have different variance trying to reject the null hypothesis that their variances are actually consistent. The statistic F is the ratio of one variance to the other, so values either >> 1 or << 1 will indicate very significant differences. A small numerical value of F implies that the two samples have significantly different variances.

The test statistics used to test the null hypothesis is:

$$F = \frac{(n-k-1)r^2}{k(1-r^2)} \tag{4.20}$$

where n is the number of data points, k is the number of coefficients in the model, excluding the intercept if any, and r^2 is the correlation of determination given by Eq.(4.18).

The F-test statistic becomes large as the coefficient of determination r^2 becomes large. To determine how large F must be before making a conclusion at a given significance level that the model is useful for predicting the dependent variable, the rejection region is set up as follows:

$$F > F\alpha\ (k, n-k-1) \tag{4.21}$$

where α is the significance level, usually $\alpha = 0.05$ or 0.1. $F\alpha$ is a two-tailed value of the F distribution that can be found from statistical tables.

4.4.1.8 T-Test

The T-test is the conventional statistic for measuring the significance of a difference of means. The significance is a numerical value between 0 and 1, and is the probability that |t| could be this large or larger just by chance, for distributions with equal means. Therefore, when the probability is less than a small numerical value (i.e. < 0.01), the independent variable is considered significant in the model. The usual t statistic for testing the equality of means of two distributions, from two independent samples can be given by:

$$t_j = \frac{b_j}{SD_j} \tag{4.22}$$

Equation 4.14 of the t-test is distributed under rejecting the null hypothesis that the parameter estimate is zero. b_j is the estimated parameter of each independent variable considered in the model. s_{Dj} is the standard error of each independent variable, estimated by:

$$s_D = \sqrt{\frac{\left[\sum(x_i - \bar{x}_1)^2 + \sum(x_i - \bar{x}_2)^2\right](1/n_1 + 1/n_2)}{(n_1 + n_2 - 1)}} \tag{4.23}$$

where, n_1 and n_2 are the number of data points for each sample. $\overline{x1}$ and $\overline{x2}$ are means of two independent samples.

4.4.2 Graphical Error Analysis

Graphical plots aid in visualizing the accuracy of an empirical equation. Among other graphical techniques are crossplot and error distribution.

4.4.2.1 Crossplot

A crossplot is a plot of the estimated values versus the corresponding experimental values. A 45° straight line is then drawn on the crossplot on which the estimated and experimental values are equal. The accuracy of an empirical equation is judged by showing closer plotted data points to the perfect 45° line.

4.4.2.2 Error Distribution

This technique involves plotting frequency of deviations versus the residual (experimental minus estimated value) histogram and then fitting a normal-distribution to it. This plot is useful in validating the adequacy of the regression model. A better empirical equation should match closer to the normal-distribution curve.

CHAPTER 5

CHAPTER 5

DEVELOPMENT OF EMPIRICAL MODELS

5.1 Experimental Data

Experimental water-oil imbibition relative permeability data from different oil fields were gathered for the development of empirical equations. The experimental data are exclusively for sandstone reservoirs ranging from semi-consolidated to unconsolidated, fine to very fine-grained, well-sorted rock types. Steady-state and unsteady-state techniques of measuring water-oil relative permeability were employed on 46 waterflood core tests in obtaining a total of 827 experimental data points. Most of the waterflood tests were conducted using the unsteady-state method. A summary of core samples' properties and test conditions used in the measurement of relative permeability values is given in Table 5.1. The obtained experimental data showed a wide range of core length, injection of displacing rate, viscosity and viscosity ratios of fluids, and temperature.

For each core test all pertinent data to fluid displacement, core properties, fluid properties, and relative permeability values were stored into the computer. Also, each set of relative permeability data was verified and analyzed individually for any discrepancies, and properly normalized following standard normalization procedures. The experimental data were checked for the capillary end effects using the criteria of Rapoport and Leas[41]. The scaling factor was calculated for each displacement test and the obtained results, shown in Table 5.1, were within the range of published experimental values. Due to the screening analysis made for all the experimental data, few relative permeability curves were found improperly generated. These curves were checked and reproduced utilizing the experimental displacement data and core properties, and using a computer program that implements the JBN method to calculate relative permeabilities from unsteady-state two-phase flow in displacement experiment.

Table 5.1: Statistical Data Description of Core Samples and Fluid Properties

Property	Min.	Max.	Mean	Std Dev
Core length, cm	4.39	31.55	14.40	10.99
Area of core, cm^2	10.01	11.58	10.96	0.340
Porosity, %	23.07	33.80	28.55	2.680
Pore volume of core, cm^3	13.80	105.19	43.32	32.05
Displacement rate, cm^3/min	1.56	9.02	6.38	2.233
Scaling factor, $L\mu v$, cm^2cp/min	1.00	4.98	2.37	0.986
Water viscosity in test, cp	0.387	1.073	0.62	0.186
Oil viscosity in test, cp	1.10	16.00	6.35	4.070
Temperature of core during flood, ^oF	74.0	165.0	133.3	28.65

Table 5.2 : Ranges of Rock and Fluid Saturation Properties

Property	Min.	Max.
Water saturation, %	11.728	93.812
Connate water saturation, %	11.728	38.556
Residual oil saturation, %	6.188	39.850
Effective oil permeability at S_{wi}, md	25.30	4790.0
Effective water permeability at S_{or}, md	4.329	1084.1

5.2 Development of Empirical Equations

Non-linear and linear multiple least-square regression analysis procedures were applied on the experimental data using SAS software package. Nonlinear and linear least-square regression Fortran programs were developed and used in this study for time saving and convenience. These programs were used in finding the best regression model out of several different forms of regression models. Analysis of variance similar to those available in SAS were implemented in these programs. However, SAS is used for final confirmation of results and further statistical analysis. The oil and water relative permeability empirical equations were derived as a function of water saturation, irreducible water saturation, residual oil saturation, oil relative permeability at irreducible water saturation, and water relative permeability at residual oil saturation. Several model selection techniques such as stepwise regression, R-square, maximum and minimum r^2 improvement, were used in selecting the best regression equations from the specified set of parameters. All these selection methods showed agreements of regression results and the stepwise regression technique was selected for the final run.

Table 5.2 shows the range of rock and fluid saturation properties of the 46 sets of imbibition water-oil relative permeability curves for sandstone reservoir rocks utilized in the regression analysis.

5.2.1 Oil Relative Permeability Model

The oil relative permeability model was derived as a function of rock and fluid saturation properties.

$$k_{ro} = f(S_w, S_{wi}, S_{or}, k_{ro(S_{wi})}) \tag{5.1}$$

The best empirical equation for estimating oil relative permeability curve was found as:

$$k_{ro} = k_{ro(Swi)} \left(\frac{1 - S_w}{1 - S_{wi}}\right)^{3.661763} \left(\frac{1 - S_w - S_{or}}{1 - S_{wi} - S_{or}}\right)^{0.706273} \tag{5.2}$$

The above equation was developed using multiple linear least-square regression applying the appropriate transformation, to generate the coefficients of the linearized model. The physical parameter, $k_{ro(Swi)}$, usually has the value of 1. The above equation satisfies exactly the requirements that:

(i) at $S_w = S_{wi}$, $k_{ro} = k_{ro(Swi)} = 1$, and

(ii) at $S_w = 1 - S_{or}$, $k_{ro} = 0$

Equation (5.2) was developed with a correlation coefficient of 0.979 indicating that 98% of the data variation about the mean is explained by the model. Analysis of the significance of the independent parameters of the model

is presented in Table 5.3. The first column shows the variable used as the regressor in the model. The second column gives the t-test statistic for rejecting the null hypothesis that the parameter is equal zero. The larger this value, the more significance of the contribution of the independent variable to the model. The Prob > |T| is the probability of the independent variable of not being significant to be added in the model. All the considered independent variables have very small values indicating their significant contributions in the model. Figure 5.1 shows the error distribution histogram constructed as the deviations frequency versus the residual (the difference of experimental and estimated value) for this study's oil relative permeability empirical equation. Most of the errors distributed closely around the mean of zero while less than 1% of data deviations occurs at the residual extremes.

Other physical parameters, not included in the model, that may affect the characteristics of oil relative permeability curve were investigated in this study. Due to the limiting data available for this work, only the effects of porosity, temperature and measurement methods of relative permeability were considered. The effect of wettability was not considered in this study because insufficient laboratory information was available to classify the data accordingly. However, most of the data used revealed characteristics for water-wet to intermediate wettability system.

Different formulations of models were attempted for several specified sets of variables (combination, cross-product, power, mathematical functions,..) and the porosity variable. All the models attempted with the porosity yielded results implying that the porosity is not a strongly significant physical parameter to be included in the oil relative permeability model.

Table 5.3: T-test for Regression Coefficients of Oil Relative Permeability Model.

Independent variable	T for H0: Parameter = 0	Prob > |T|
$Ln\left(\frac{1 - S_w}{1 - S_{wi}}\right)$	24.655	0.0001
$Ln\left(\frac{1 - S_w - S_{or}}{1 - S_{wi} - S_{or}}\right)$	10.896	0.0001

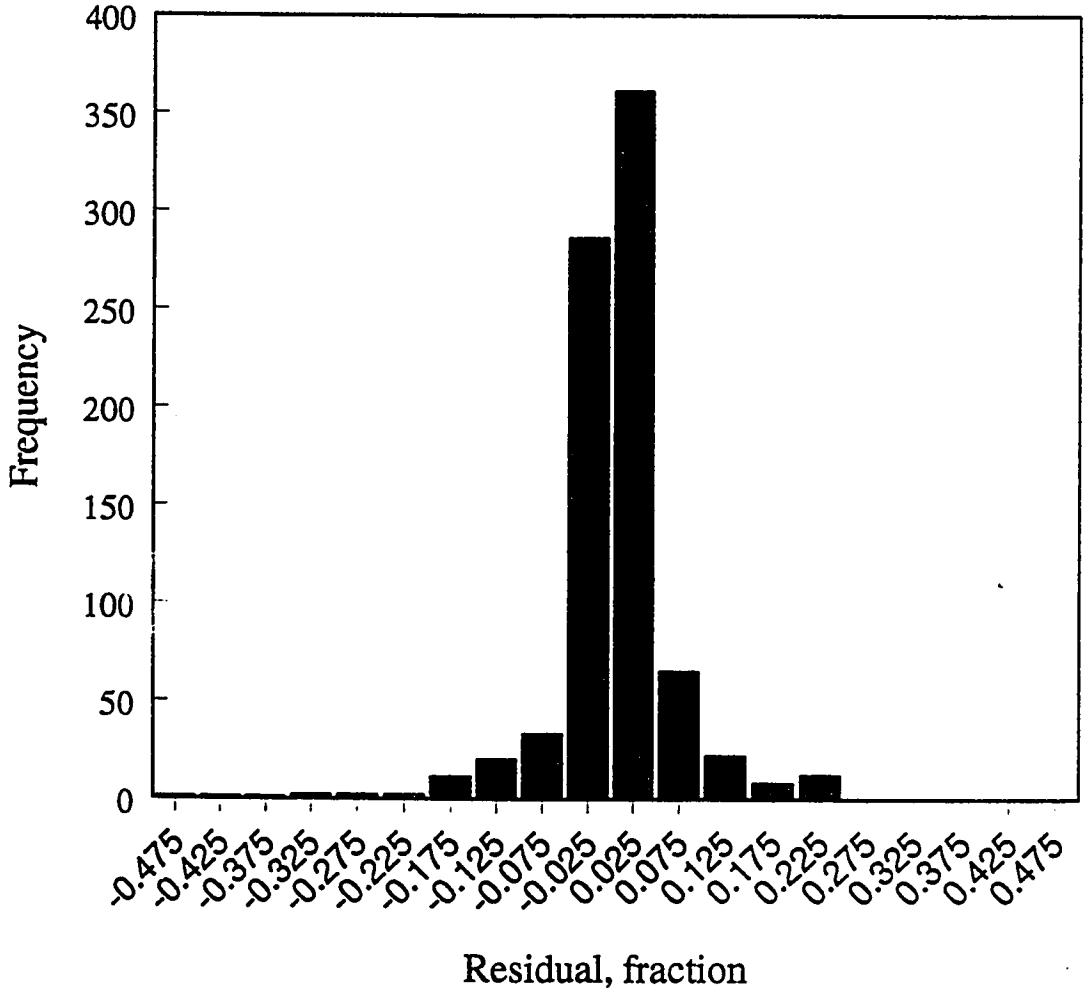


Figure 5.1- Error Distribution Plot for Oil Relative Permeability Empirical Equation (This study)

The experimental data were classified in term of temperature to study its effect on the prediction of oil relative permeability curve. Because few tests were conducted at surface temperature, the experimental data were classified with a temperature range of 120 to 165 °F with a total of 779 data points from 41 displacement core tests. These data were used in the developed regression model to generate its correlating coefficients. The statistical analysis showed no improvement to the oil relative permeability model by classifying the data according to the temperature. This indicates that the oil relative permeability is independent of temperature for the data used in this study.

The experimental relative permeability data were also classified in terms of the laboratory methods used to produce them, for evaluating their influence on determining oil relative permeability curves. The laboratory data collected in this study were measured using steady-state and unsteady-state methods. A total of 799 data points from 42 waterflood tests was measured using unsteady-state method. These data were reproduced using the developed regression model of oil relative permeability. Analyses of regression and statistical results of the model indicated that classifying the experimental data according to the method of measurement does not contribute significantly to improvement of predicting oil relative permeability.

5.2.2 Water Relative Permeability Model

The water relative permeability model was developed as a function of fluid saturations and porosity following the basic concept of Chierici model [8] for water relative permeability.

$$k_{rw} = f(S_w, S_{wi}, S_{or}, k_{rw(s_{or})}) \tag{5.3}$$

The following empirical equation was found to reproduce accurately the experimental water relative permeability data:

$$k_{rw} = Exp[-5.41463\phi(1-S_{or})] \cdot Exp\left[-1.05086\left(\frac{S_w - S_{wi}}{1-S_w-S_{or}}\right)^{-0.46163}\right] \tag{5.4}$$

Nonlinear least-square regression was applied on the experimental data to generate the correlating coefficients of the model. The first exponential term of equation (5.4) represents very accurately the end point of water relative permeability curve, Eq.(5.5). It was derived as a function of residual oil saturation and porosity and it individually contributes for more than 54% of the improvement of accuracy of the entire model. The water relative permeability curve can be normalized by dividing the water relative permeability data by this term. Equation (5.5) was developed with a correlation coefficient of 0.919 using a linear least-square regression analysis.

$$k_{rw(s_{or})} = Exp\left[-5.41463\phi(1-S_{or})\right] \quad (5.5)$$

The water relative permeability model implicitly satisfies the initial and end points of water relative permeability curves:

(i) At $S_w = S_{wi}$, $\dfrac{S_w - S_{wi}}{1 - S_w - S_{or}} = 0$, and $e^{-\infty} = 0$

implying that $k_{rw} = 0$

(ii) At $S_w = 1 - S_{or}$, $\dfrac{S_w - S_{wi}}{1 - S_w - S_{or}} = \infty$, and $e^{o} = 1$,

implying that $k_{rw} = Exp[-5.41463\phi(1-S_{or})]$

The empirical equation obtained using this model agrees very closely with experimental data. Equation (5.4) was developed with a correlation coefficient of 0.9304 implying that 93% of the data variation around the zero mean is accounted for by the model. The significance F-test statistic for this model is 2433.0 with " Prob > F " value of 0.0001 indicates that all the independent variables included in the considered model contribute significantly to the improvement of the model. Figure 5.2 shows the error distribution histogram constructed as the deviations frequency vs. the residual for water relative permeability empirical equation of this study. Most of the errors distributed closely around the mean of zero while less than 1% of data deviations occurs at the residual extremes.

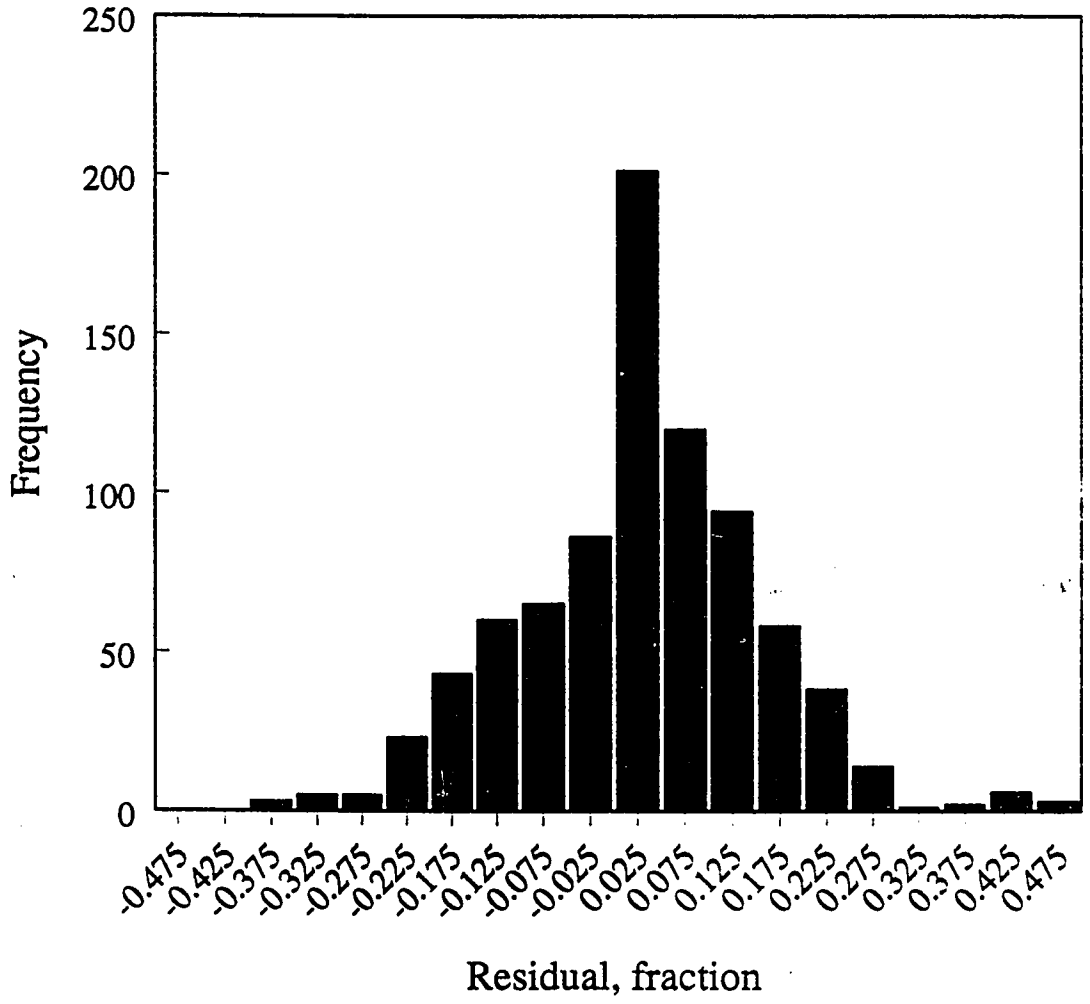


Figure 5.2- Error Distribution Plot for Water Relative Permeability Empirical Equation (This study)

Similar to the work done for oil relative permeability model, the effect of other physical parameters on the characteristics of water relative permeability curves were investigated. Unlike for oil relative permeability model, the porosity was found a strong independent variable to be included in water relative permeability model particularly in estimating the end point of water relative permeability curve. This implies that the porosity has an influence on the characteristics of water relative permeability.

Classification of experimental data, as mentioned in the previous section, according to the temperature indicated that the water relative permeability is slightly influenced by temperature for the limited data used in this study.

The classified experimental data with unsteady-state method were reproduced using the developed regression model of water relative permeability. It was found that classifying the experimental data according to the method of measurement has no significant effect on the improvement of predicting water relative permeability.

5.2.3 Behavior of Models

It is important that the dependent variables of regression models comply with the behavior of their independent correlating variables. The behaviors of regression models of this study were examined against their correlating parameters of physical properties. The average experimental values of irreducible water saturation, residual oil saturation, and porosity were used to calculate water and oil relative permeabilities from regression models for the

range of water saturation. Figure 5.3 depicts a typical behavior of water-oil relative permeability curves as generated by the models of this study. For the same case, a semilog plot of water-oil relative permeability curves is shown in Figure 5.4.

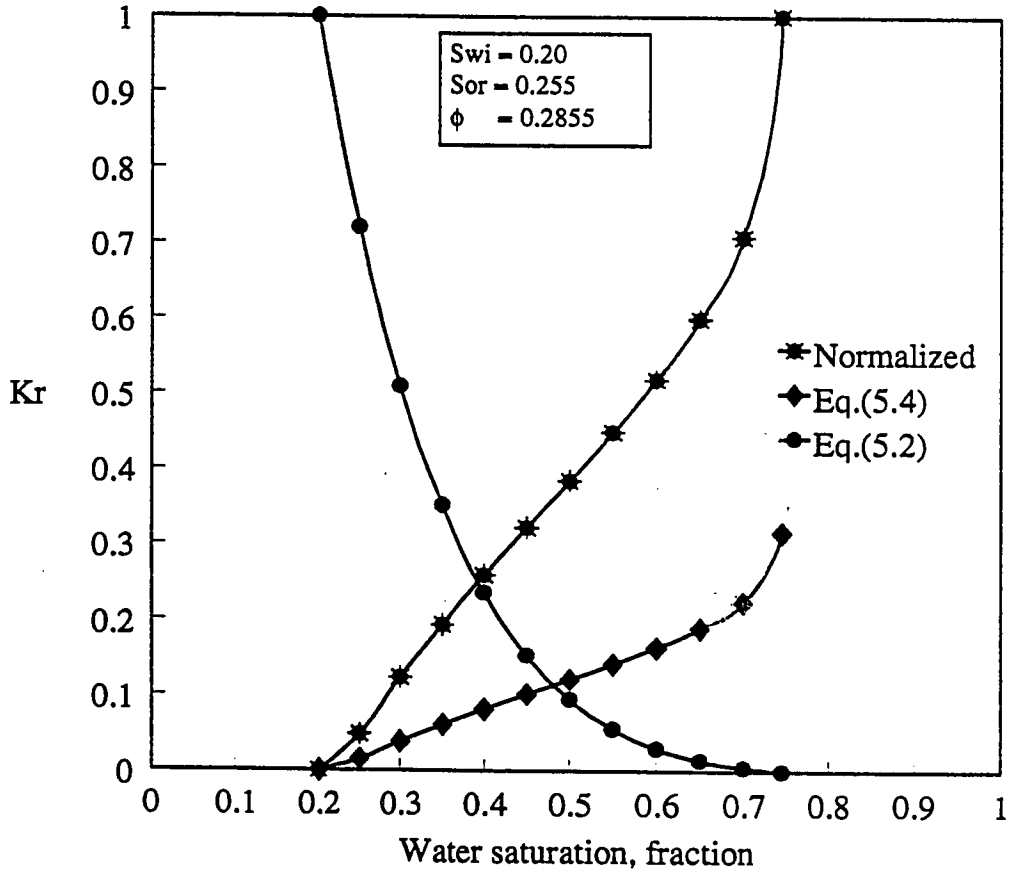


Figure 5.3- Behavior of Water-Oil Relative Permeability Models Against their Physical Correlating Properties.

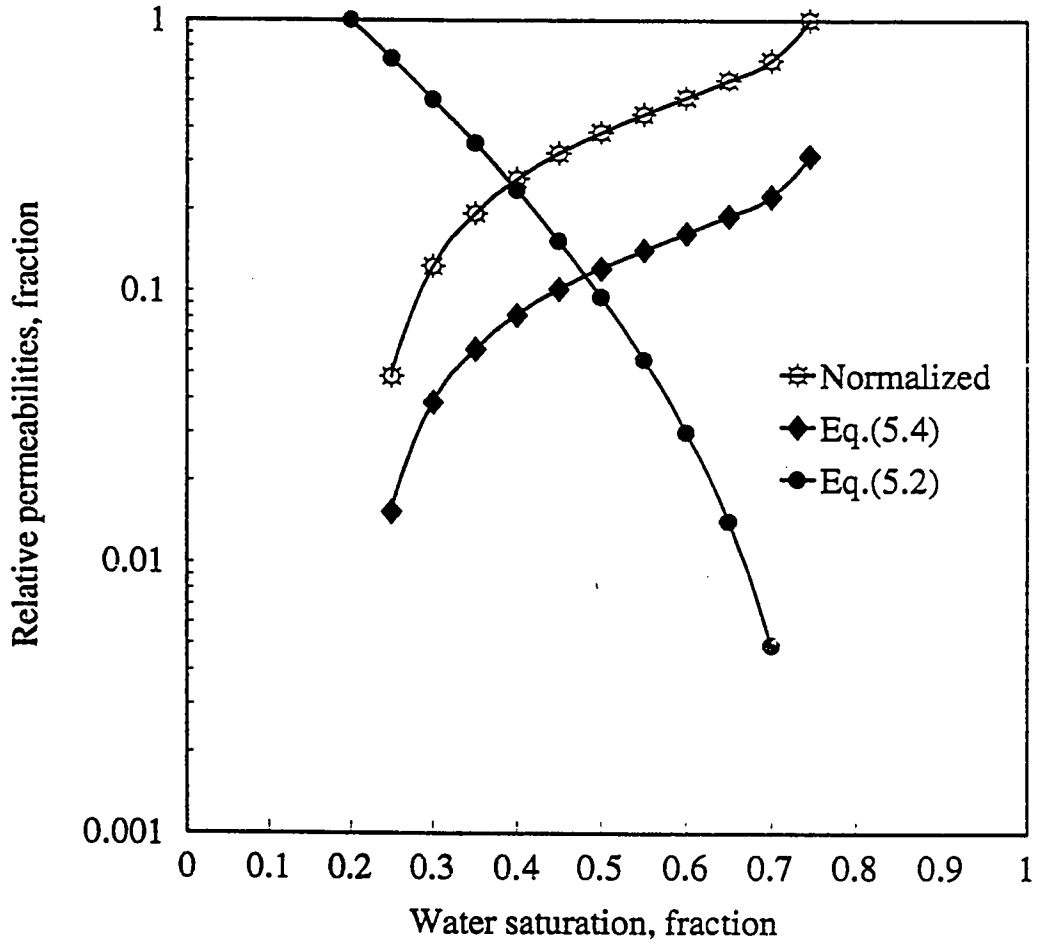


Figure 5.4- Semilog Plot of Water-Oil Relative Permeability from Empirical Equations

CHAPTER 6

CHAPTER 6

COMPARATIVE EVALUATION OF PUBLISHED EMPIRICAL EQUATIONS

The new developed empirical equations were evaluated against published empirical equations for checking their performance and degree of accuracy in estimating the water-oil relative permeability curves. Two error analysis techniques were used: statistical and graphical error analysis.

6.1 Statistical Error Analysis

The statistical parameters used to compare the degree of accuracy of the water-oil relative permeability empirical equations are: average error, absolute average error, standard deviation, root-mean-square, minimum and maximum absolute average error, and the F-test statistic. These parameters were computed for the published empirical equations using the 827 experimentally obtained data points. Table 6.1 shows the statistical accuracy of oil relative

permeability empirical equations. The empirical equation of this study achieved the lowest errors, standard deviation and root-mean-square error, with the highest F-test statistic. Empirical equation of Honarpour et al. stood second in the accuracy of errors but with lower F-test statistic than Wyllie's empirical equation. Naar et al.'s empirical equation showed poor accuracy, with the highest errors and the lowest F-test statistic.

The statistical accuracy of empirical equations for water relative permeability is given in Table 6.2. This study again achieved the highest accuracy of estimating water relative permeability curves with the lowest errors and highest F-test statistic. Honarpour et al.'s empirical equation for oil-wet and intermediate wettability stood second in the accuracy of errors but with unsatisfactory F-test statistic.

Naar et al., Jones and Land have all the same empirical equation and hence have the same unsatisfactory performance of accuracy. Wyllie's empirical equation showed similar performance of errors to Honarpour et al.'s empirical equation for oil-wet and intermediate wettability, but with better F-test statistic. Pirson's empirical equation showed poor performance of accuracy with the highest errors, and Honarpour et al.'s empirical equation (for water-wet and intermediate) yielded the lowest F-test statistic.

6.2 Graphical Error Analysis

The crossplot graphical error analysis technique is used in the evaluation of performance of this study and other published empirical equations.

Figures 6.1 through 6.7 show the crossplots of estimated vs. measured values for oil relative permeability empirical equations. Most of the plotted points of this study's empirical equation fall very close to the perfect 45° straight line. All other published empirical equations overestimate the experimental data of oil relative permeabilities with high deviations from the perfect 45° line.

Crossplots for water relative permeability empirical equations are shown in Figures 6.8 through 6.13. The closeness of the data points to the perfect 45° line for this study's empirical equation is obvious. All other published empirical equations reveal their underestimation of the experimental data of water relative permeability.

Table 6.1: Statistical Accuracy of Oil Relative Permeability Empirical Equations

Eq.	Researcher(s)	Year	E_{av}	E_{ab}	s^2	rms	E_{max}	F-value
2.2	Wyllie	1951	-0.097	0.099	0.017	0.131	0.559	3160.0
2.9	Pirson	1958	-0.133	0.139	0.023	0.153	0.345	675.7
2.13	Naar et al.	1961	-0.220	0.220	0.067	0.259	0.701	450.4
2.16	Jones	1966	-0.151	0.151	0.034	0.184	0.594	644.3
2.18	Land	1968	-0.088	0.089	0.018	0.134	0.592	548.4
2.24	Honarpour et al.	1982	-0.057	0.064	0.008	0.090	0.535	1622.2
5.2	This study	1994	-0.001	0.036	0.004	0.066	0.481	9028.8

Table 6.2: Statistical Accuracy of Water Relative Permeability Empirical Equations

Eq.	Researcher(s)	Year	E_{av}	E_{ab}	s^2	rms	E_{max}	F-value
2.1	Wyllie	1951	0.384	0.385	0.190	0.436	0.886	692.6
2.8	Pirson	1958	0.506	0.506	0.334	0.577	0.999	317.2
2.12	Naar et al.*	1961	0.438	0.438	0.244	0.493	0.945	672.4
2.22	Hanarpour et al.	1982	0.454	0.454	0.268	0.516	0.932	128.1
2.23	Honarpour et al.	1982	0.366	0.367	0.176	0.418	0.830	169.8
5.4	This study	1994	0.023	0.070	0.010	0.101	0.460	2433.0

* Including equations of Jones (2.15), and Land (2.17).

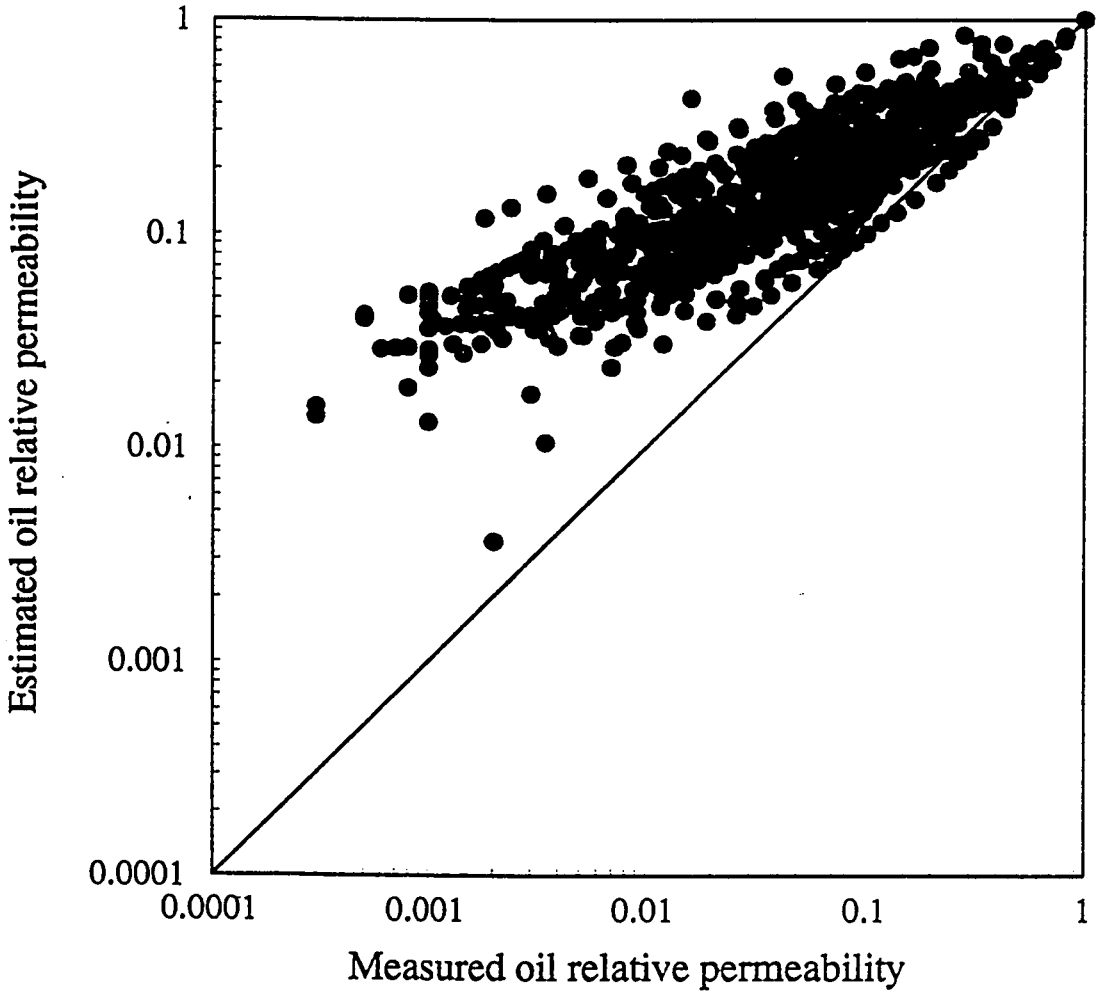


Figure 6.1- Crossplot for oil relative permeability empirical equation (Wyllie)

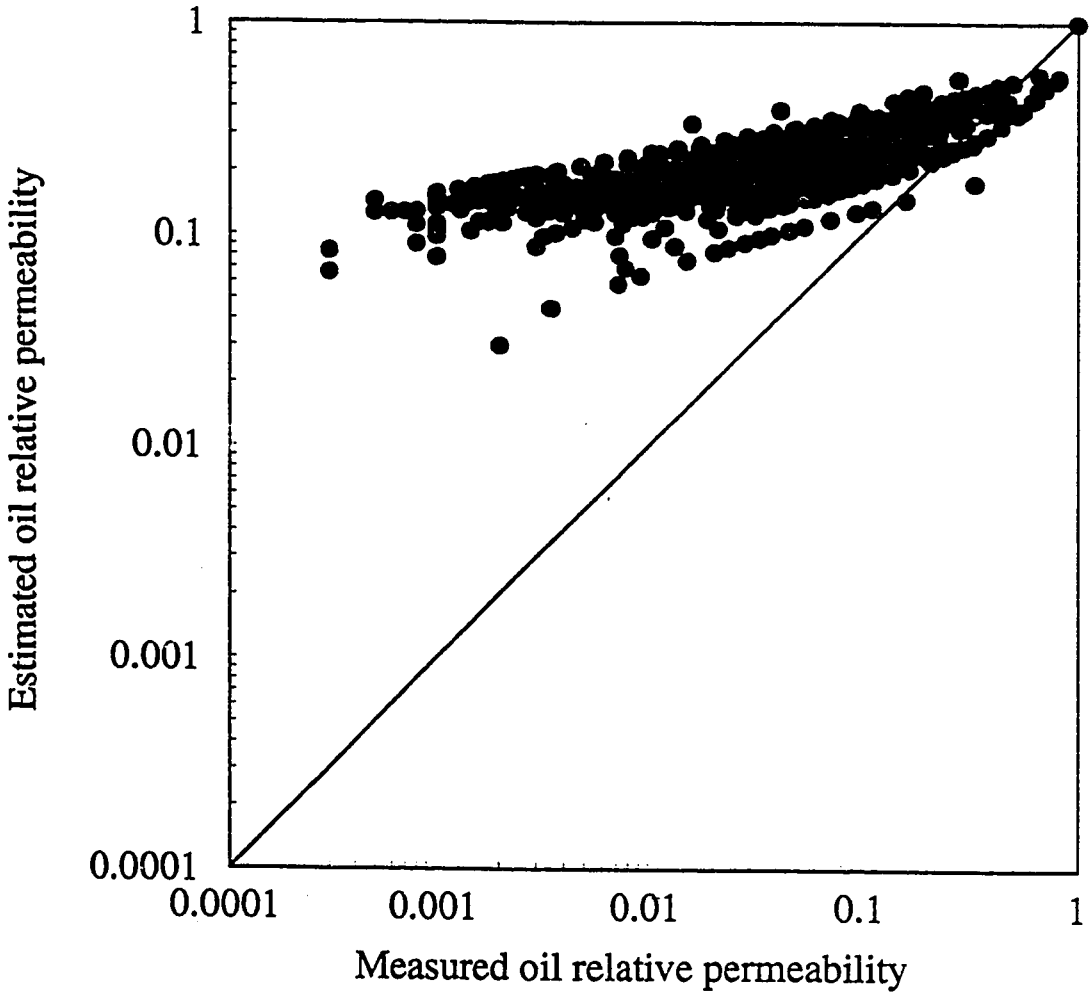


Figure 6.2- Crossplot for oil relative permeability empirical equation (Pirson)

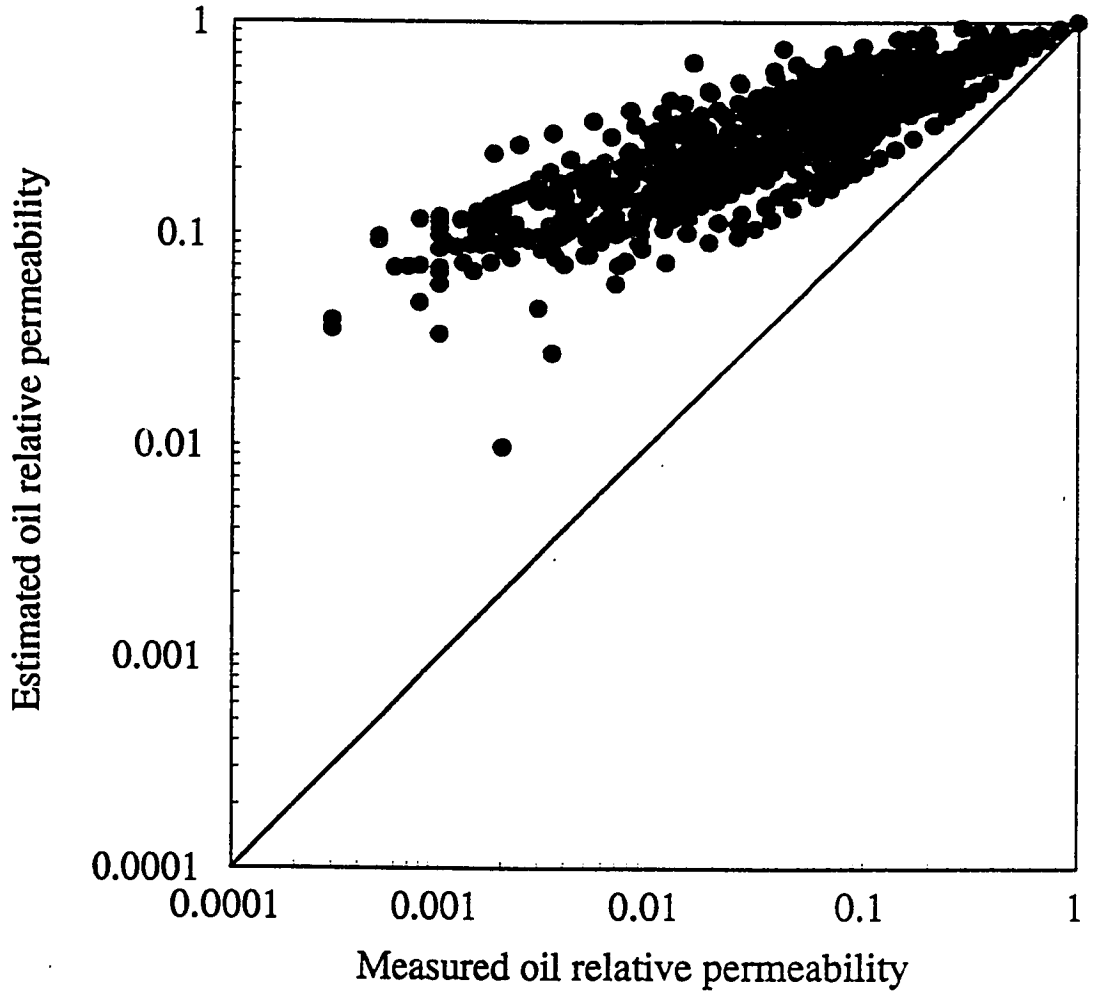


Figure 6.3- Crossplot for oil relative permeability empirical equation (Naar et al.)

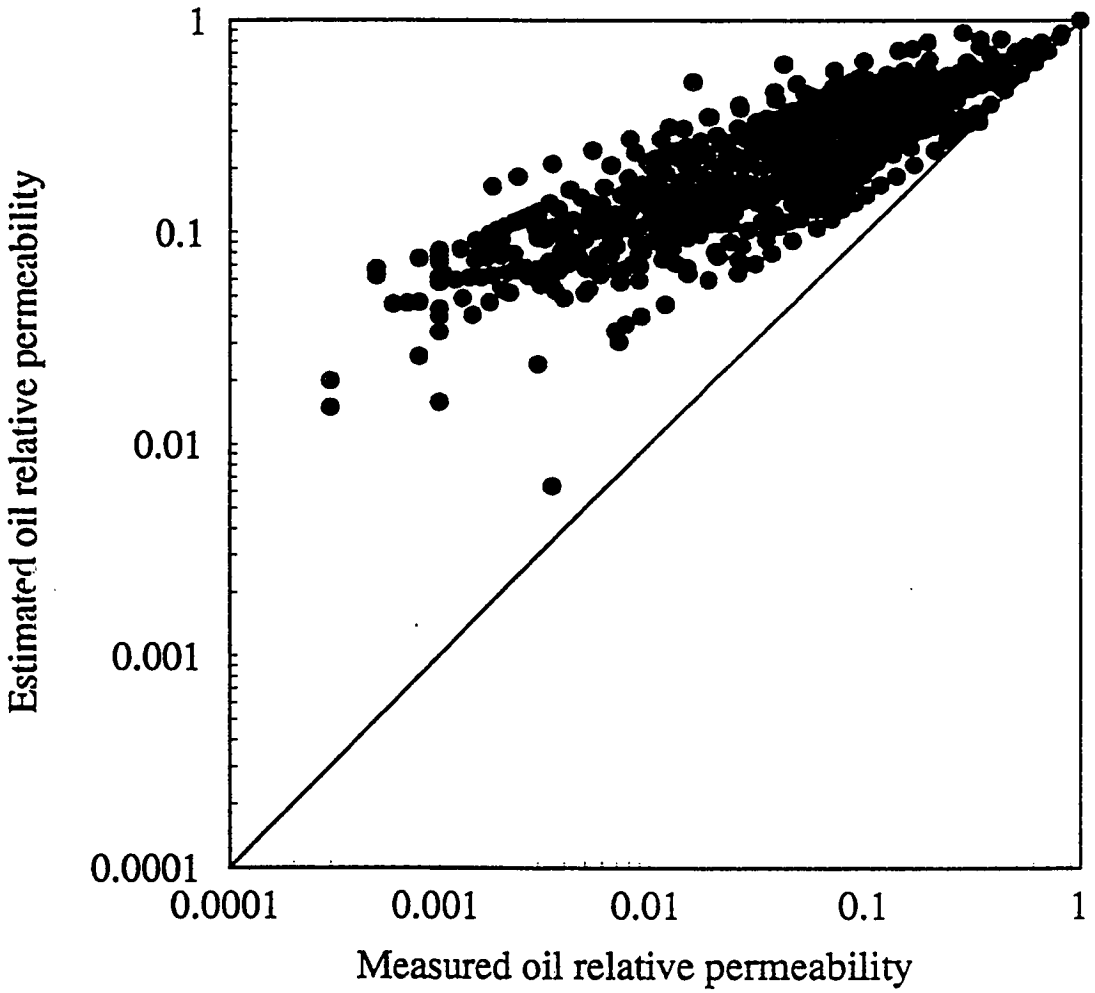


Figure 6.4- Crossplot for oil relative permeability empirical equation (Jones)

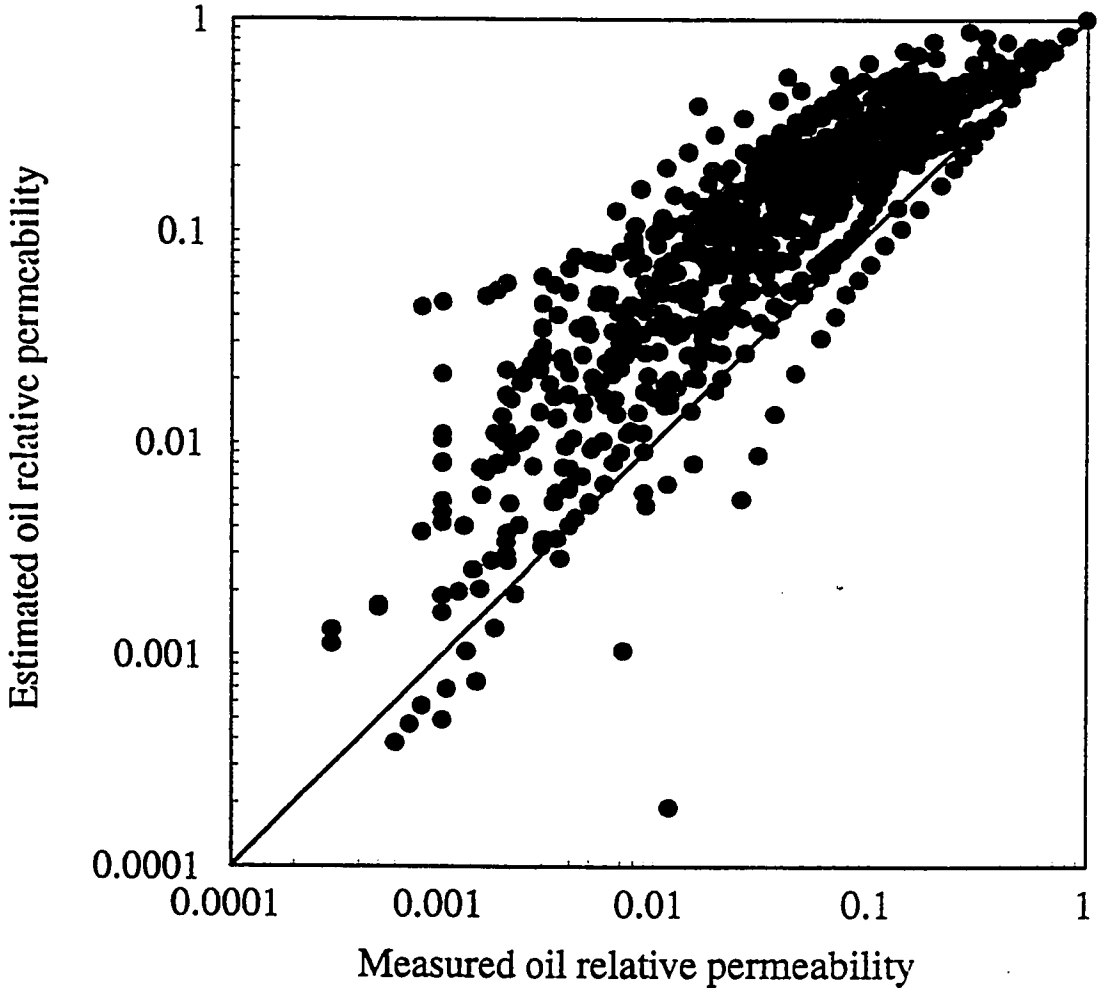


Figure 6.5- Crossplot for oil relative permeability empirical equation (Land)

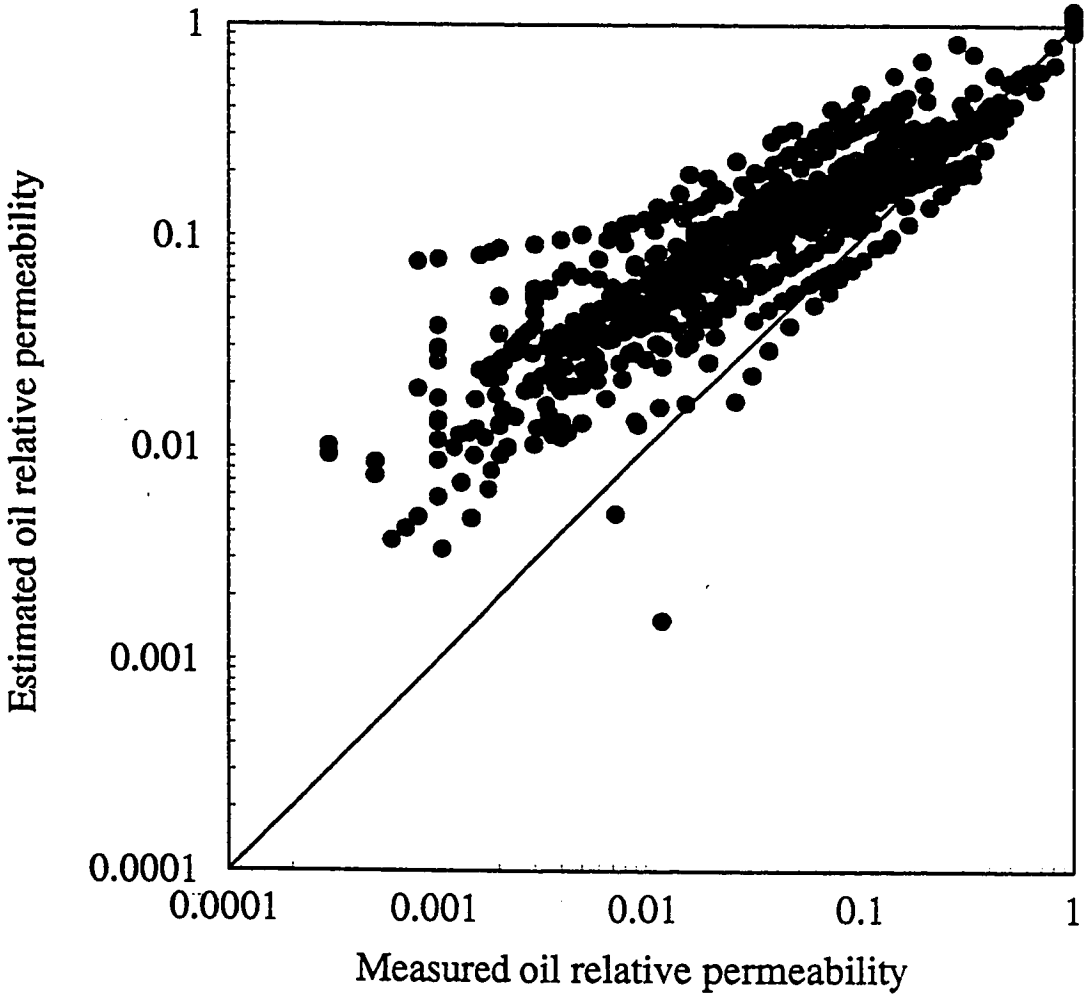


Figure 6.6- Crossplot for Oil Relative Permeability Empirical Equation (Honarpour et al.)

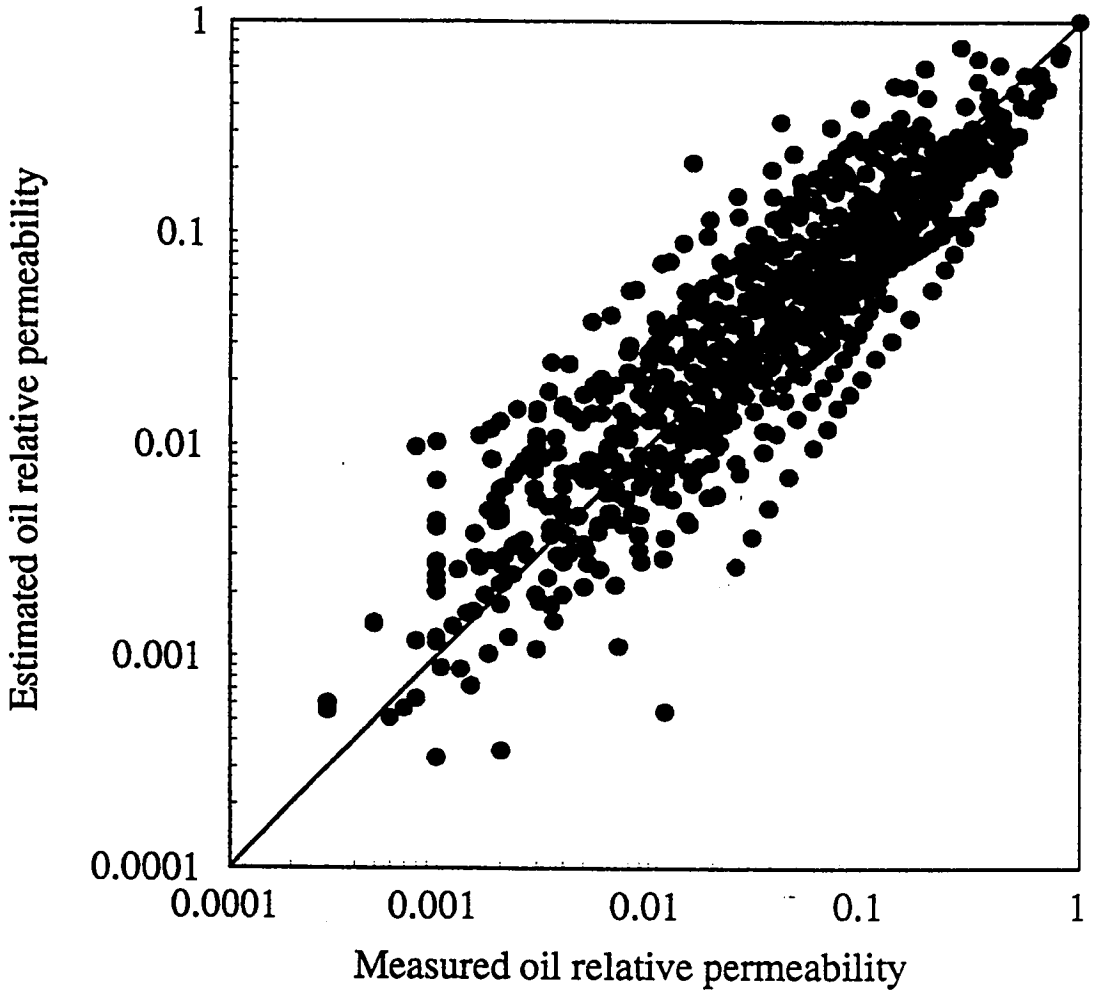


Figure 6.7- Crossplot for Oil Relative Permeability Empirical Equation
(This study)

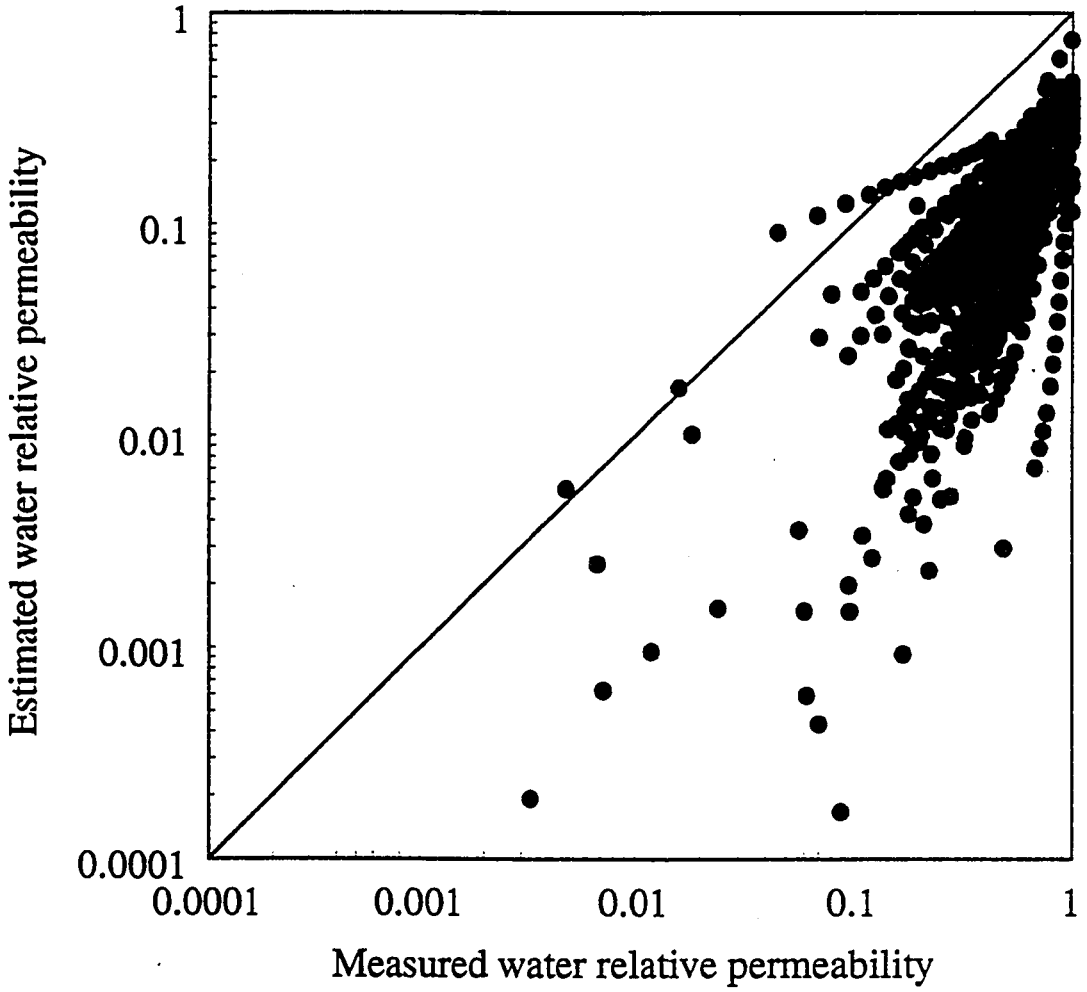


Figure 6.8- Crossplot for normalized water relative permeability (Wyllie)

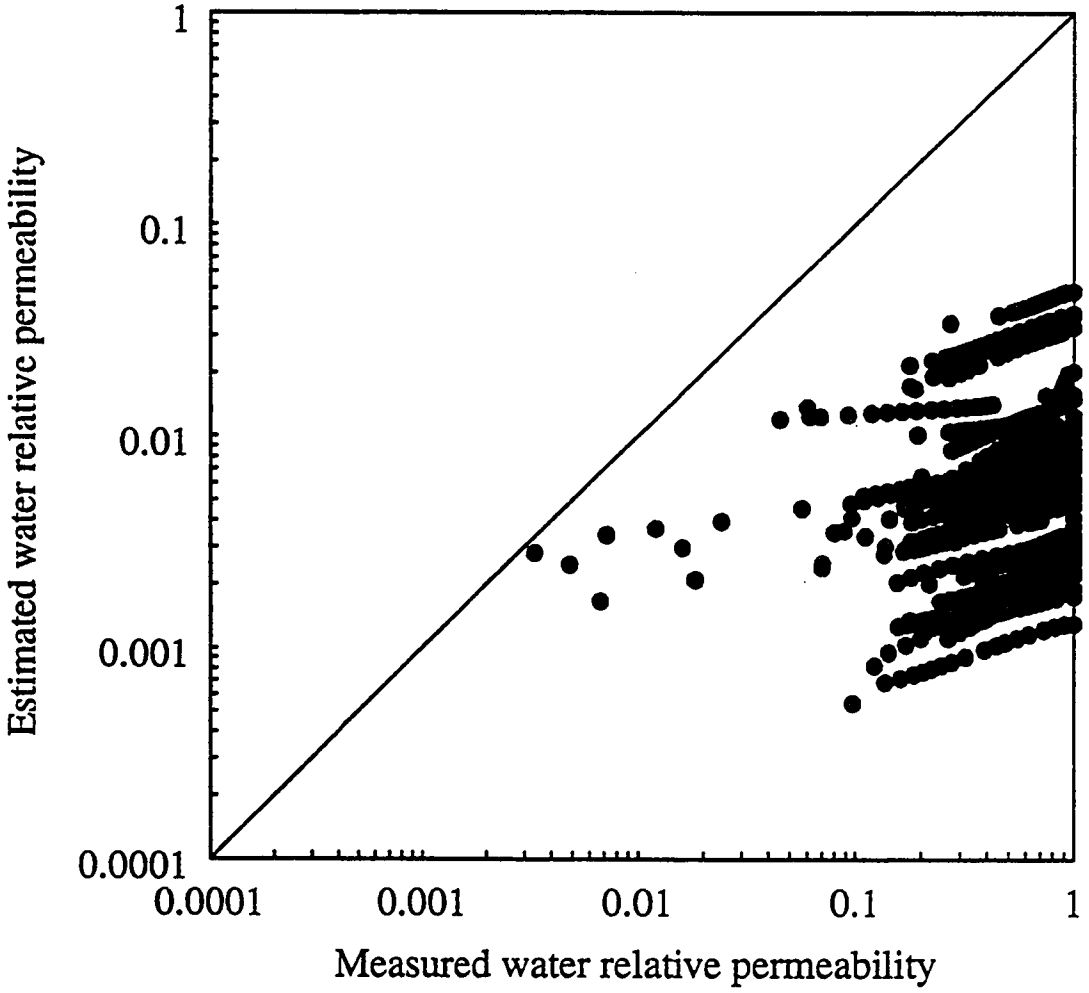


Figure 6.9- Crossplot for normalized water relative permeability (Pirson)

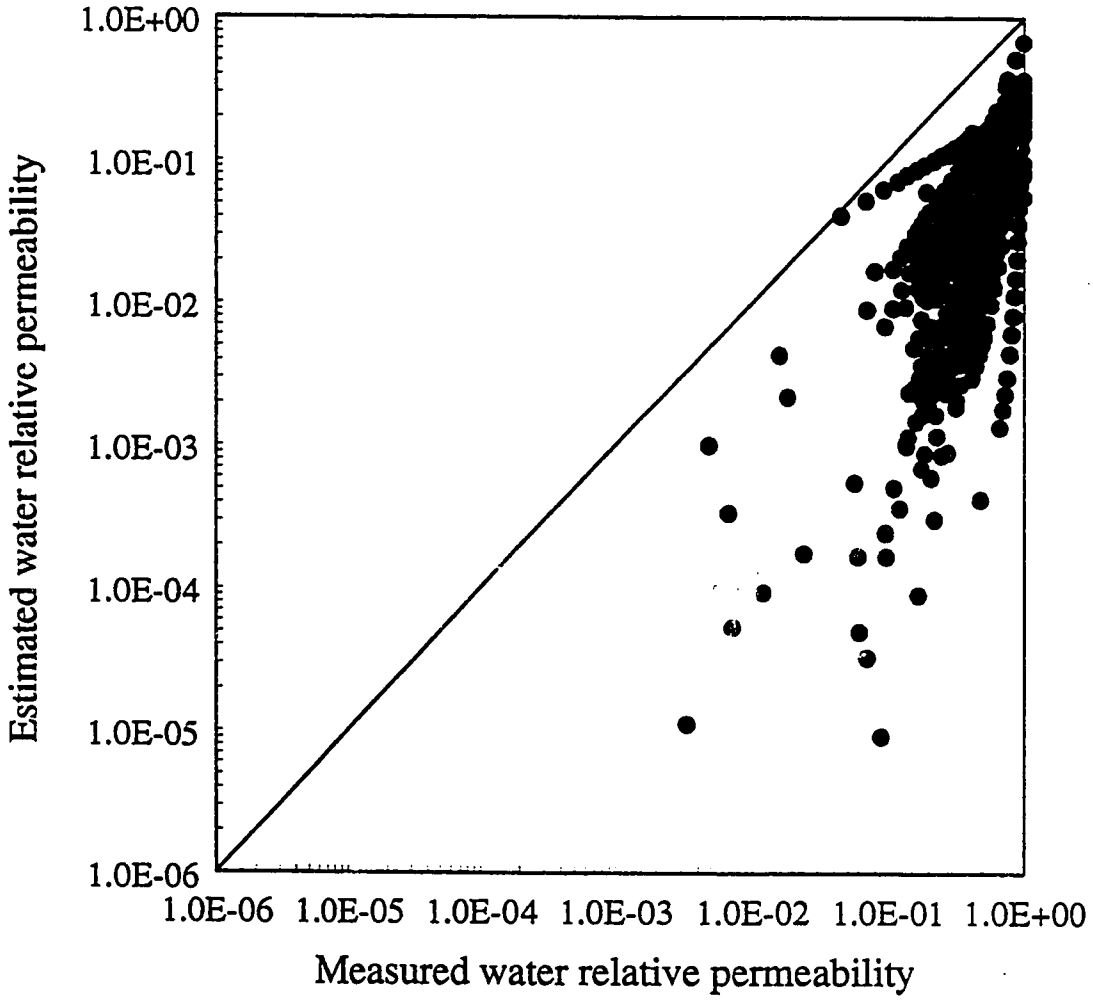


Figure 6.10- Crossplot for normalized water relative permeability
(Naar et al., Jones, and Land)

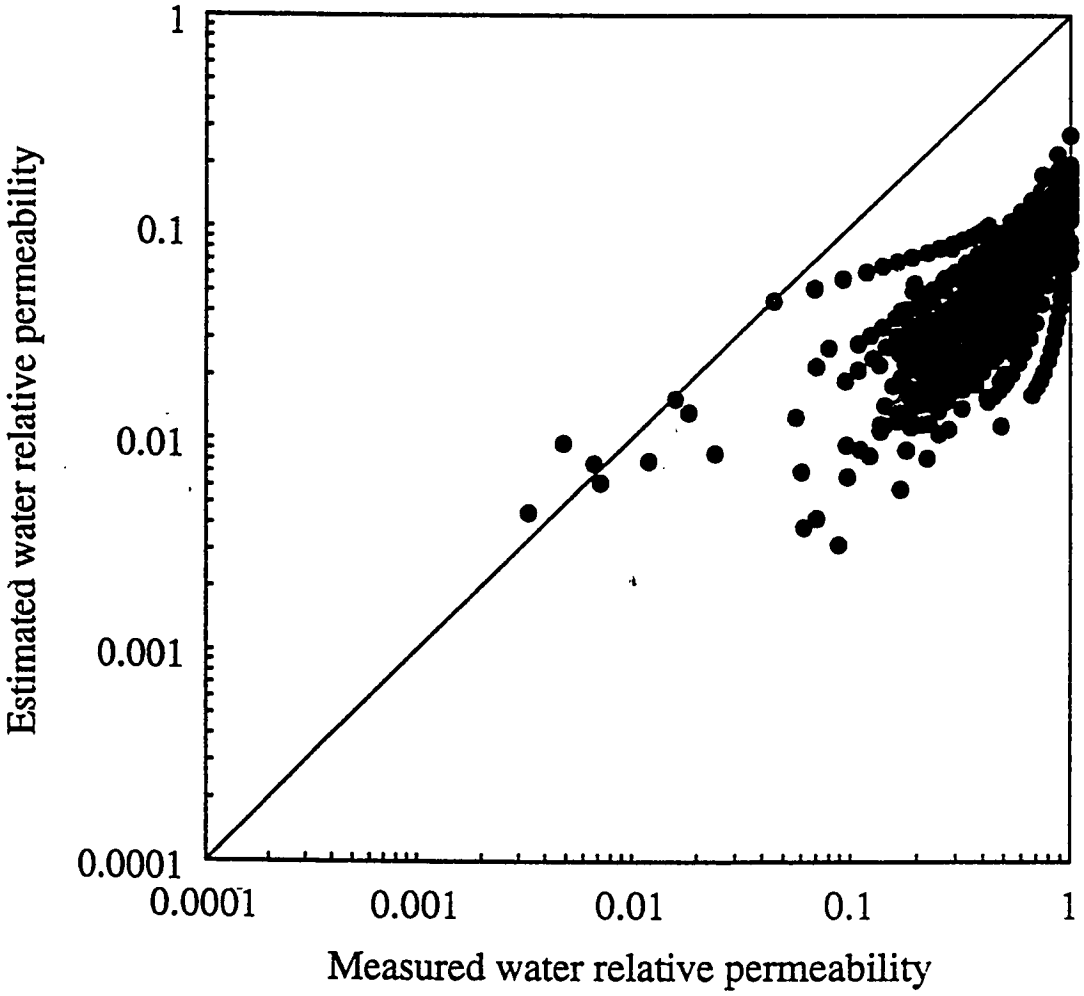


Figure 6.11- Crossplot for normalized water relative permeability (Honarpour et al., Eq. 2.22)

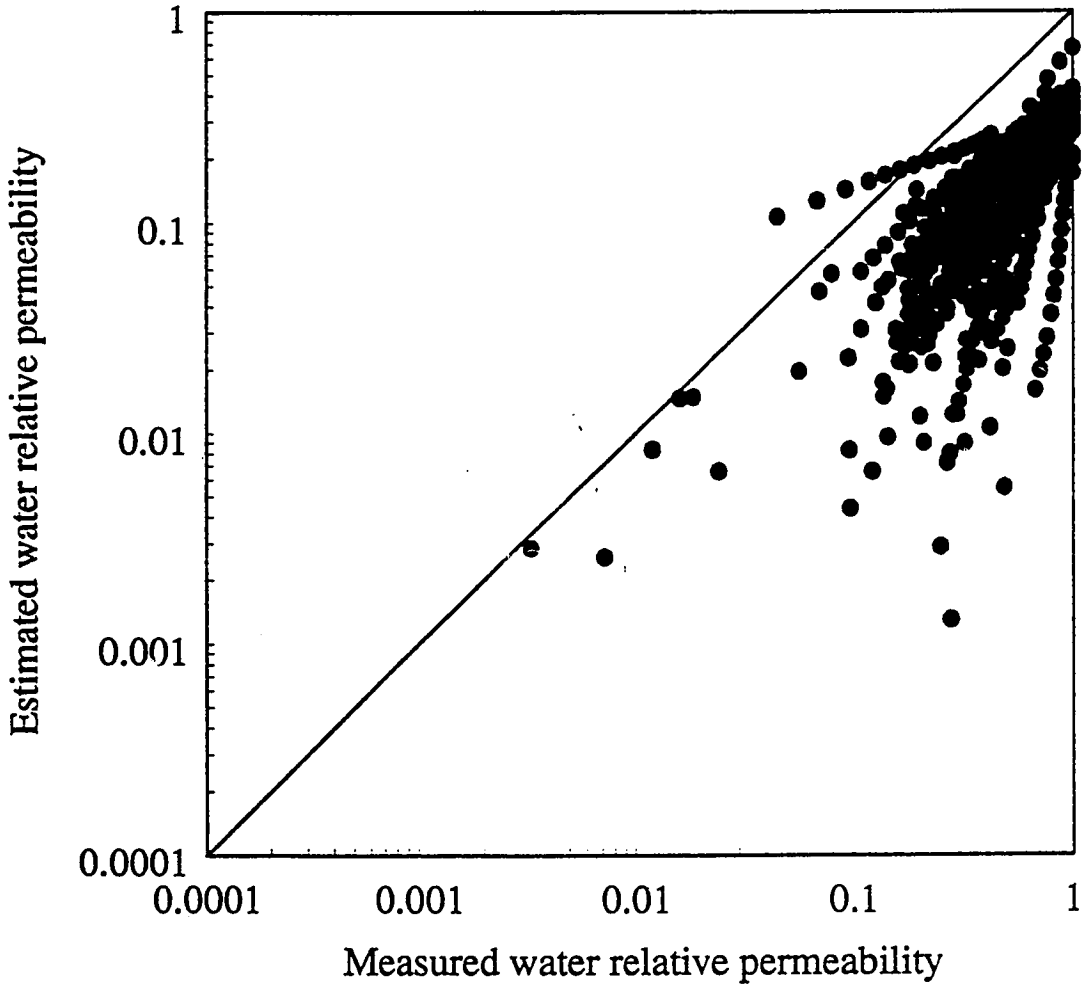


Figure 6.12- Crossplot for normalized water relative permeability
(Honarpour et al., Eq. 2.23)

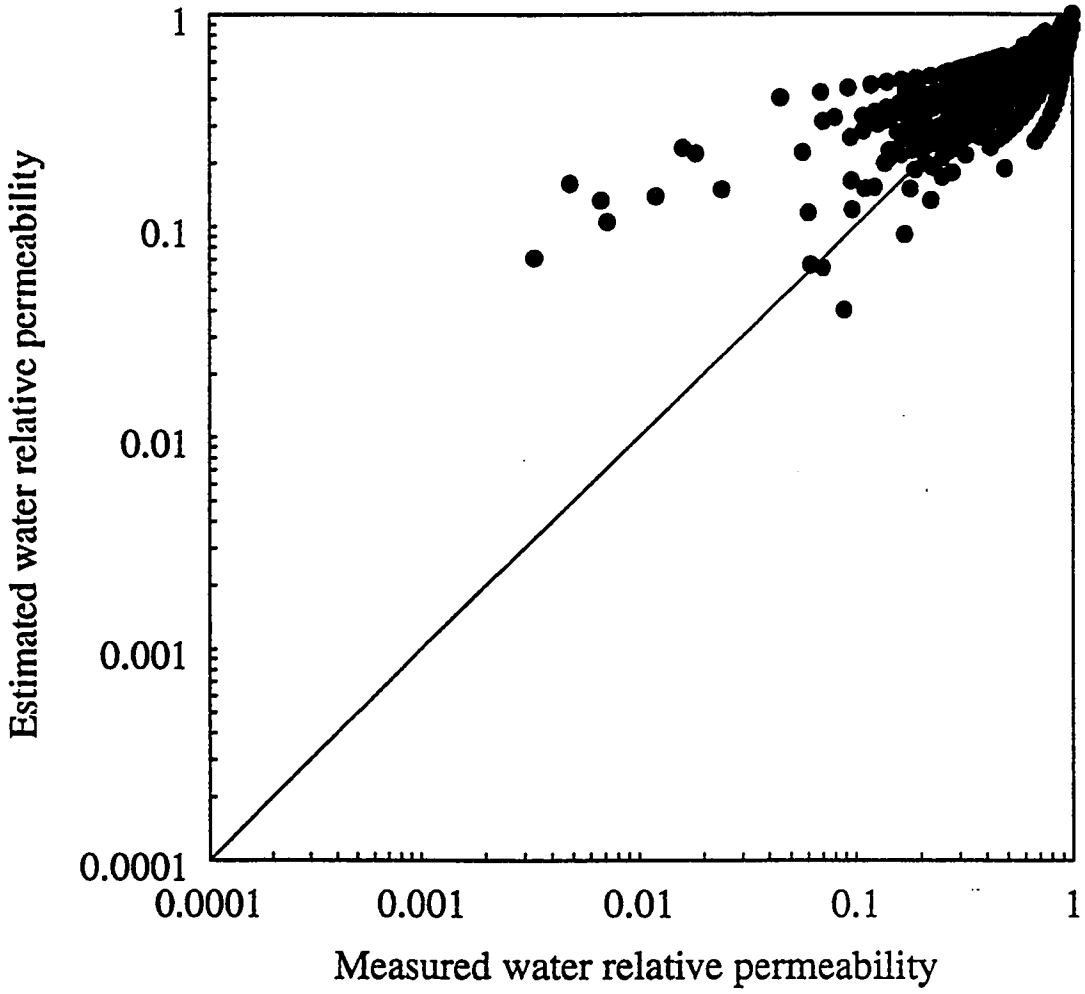


Figure 6.13- Crossplot for normalized water relative permeability (This study)

CHAPTER 7

CHAPTER 7

VALIDATION OF NEW EMPIRICAL EQUATIONS

The new developed empirical equations for water-oil imbibition relative permeability calculations were validated using published relative permeability data in the literature that were not utilized in the models developed in this study. This procedure was undertaken to examine the applicability of the new empirical equations and evaluate their performance of accuracy against previously published empirical equations. A total of 45 data points from 4 published sets[53,3,21] of water-oil relative permeability was used for validation of empirical equations. These data are summarized in Table 7.1.

The new empirical equations were compared with published empirical equations of Wyllie, Pirson, Naar et al., Jones, Land, and Honarpour et al. Each set of published data was used for comparison of published empirical equations with those developed in this study. The results showed that the new empirical equations reproduced very accurately experimental relative permeability data

than other published equations. Table 7.2 shows results of statistical accuracy for oil relative permeability models using the compiled published data sets. Empirical equations of oil relative permeability were compared in terms of average error, average absolute error, standard deviation, maximum absolute error, and coefficient of determination. The oil relative permeability model of this study yields better results of accuracy than other published models for oil relative permeability by showing lower values of errors and higher coefficient of determination. Empirical equations of Pirson and Honarpour et al. show similar performance of accuracy; however, Pirson's equation has a higher coefficient of determination than Honarpour's et al. does. Empirical equation of Naar et al. performs poorly compared to other empirical equations.

Table 7.3 gives statistical accuracy of results for water relative permeability models using compiled sets of published data. Empirical equations of water relative permeability were compared in terms of average error, average absolute error, standard deviation, maximum absolute error, and F-test statistic. Inclusion of r^2 parameter in this error analysis was impractical because small values of F-test given by empirical equations of Honarpour et al. and Pirson resulted in negative r^2 values. It should be noted that the F-test statistic has a strong direct relation with the coefficient of determination. The highest

performance of accuracy is achieved by this study which gives lower errors and higher F-test statistic than the results obtained for other studies.

Table 7.1: Summary of Published Water-Oil Relative Permeability Data Used for Validation of Empirical Equations

Source	Field	Number of Data Points	Number of Data Sets
Willhite	Chesney MP-4	28	2
Braun and Blackwell	Berea	8	1
Jones and Roszelle	-	9	1

Table 7.2: Statistical Accuracy of Oil Relative Permeability Empirical Equations

Eq.	Researcher(s)	Year	E_{av}	E_{ab}	s^2	rms	E_{max}	r^2
2.2	Wyllie	1951	-0.137	0.137	0.027	0.160	0.297	0.695
2.9	Pirson	1958	-0.104	0.126	0.023	0.146	0.295	0.913
2.13	Naar et al.	1961	-0.276	0.276	0.103	0.307	0.505	0.117
2.16	Jones	1966	-0.193	0.193	0.052	0.218	0.377	0.435
2.18	Land	1968	-0.085	0.091	0.021	0.134	0.334	0.787
2.24	Honarpour et al.	1982	0.006	0.053	0.009	0.089	0.391	0.906
5.2	This study	1994	-0.010	0.041	0.005	0.069	0.226	0.941

Table 7.3: Statistical Accuracy of Water Relative Permeability Empirical Equations

Eq.	Researcher(s)	Year	E_{av}	E_{ab}	s^2	rms	E_{max}	F-value
2.1	Wyllie	1951	0.045	0.052	0.006	0.075	0.207	72.0
2.8	Pirson	1958	0.115	0.115	0.028	0.162	0.453	10.5
2.12	Naar et al.*	1961	0.083	0.083	0.013	0.113	0.301	33.6
2.22	Hanarpour et al.	1982	0.082	0.088	0.017	0.122	0.357	4.1
2.23	Honarpour et al.	1982	0.002	0.049	0.006	0.071	0.182	18.9
5.4	This study	1994	0.000	0.018	0.001	0.026	0.065	367.7

* Including equations of Jones (2.15), and Land (2.17).

CHAPTER 8

CHAPTER 8

CONCLUSIONS

Multiple linear and nonlinear least-square regression analyses were applied on collected laboratory experimental data for development of empirical equations for water-oil relative permeability. The following conclusions were reached as a result of this study:

1. New empirical models are made available for the oil industry for calculating accurately imbibition water-oil relative permeability curves of sandstone reservoir rocks.

2. The newly developed empirical equations satisfy exactly the initial and end points requirements of water-oil relative permeability curves. The endpoint water relative permeability empirical equation developed in this study was found to reproduce accurately experimental values.

3. Comparative evaluation of existing empirical equations were made and the new empirical equations outperform all other published work for the data used in this study.

4. The empirical equations developed in this study were validated using published relative permeability data and this study showed better results of accuracy than previously published studies.

5. The new empirical equations can be readily implemented in any reservoir simulator with two-phase option.

6. Physical parameters that may affect the performance of water-oil relative permeability were investigated using parametric statistical analysis. It was found that the porosity is not a strong independent variable to be included in oil relative permeability model but it has a strong relation with water relative permeability. The temperature was found not to influence oil relative permeability but it has a slight influence on water relative permeability. Classification of data according to measurement techniques does not affect significantly the performance of both oil and water relative permeabilities.

APPENDIX

APPENDIX

EXAMPLE CALCULATION OF JBN METHOD

A constant-rate waterflood in a sucrosic dolomite core plug was performed to determine oil-water relative permeability curves. Cumulative water injection, oil production, and overall pressure-drop data are given in Table A.1. These data were taken from a much larger data set and were slightly smoothed. Other pertinent data are given in Table A.2. With the Johnson-Bossler-Naumann technique, find the k_{ro} and k_{rw} curves.[53]

The steps used to estimate the relative permeabilities are summarized as follows:

1) Calculate S_w from Eq. (3.5)

2) Calculate S_w from Eq. (3.4)

3) Calculate I_r using Eq. (3.6)

4) Estimate derivatives of (dS_w/dQ_i) and $[d(\frac{1}{Q_i I_r})/d(\frac{1}{Q_i})]$

5) Calculate k_{ro} using Eqs. (3.3) and (3.7)

6) Calculate k_{rw} using Eq. (3.8)

The results of computational procedures of JBN method for this problem are given in Table A.3. Computed relative permeabilities and saturations are plotted as shown in Fig. A.1. The curves begin at $S_w = 0.276$ where the value of $k_{rw} = 0$ and $k_{ro} = 1.0$ (the base permeability is the oil permeability at connate water saturation).

Table A.1: Waterflood Data for Example Calculation of JBN Method (Ref. 53).

Q_i, cm^3	Q_o, cm^3	dP, psi
0.00	0.00	16.31
1.51	1.51	15.93
3.02*	3.02	15.55
3.87	3.18	14.03
6.00	3.48	11.94
7.69	3.65	11.18
9.09	3.77	10.71
13.81	4.10	9.60
20.00	4.42	8.66
32.20	4.80	7.42
48.40	5.02	6.48
100.00	5.25	5.22
200.00	5.36	4.51
400.00	5.42	4.06

* Water breakthrough

Table A.2: Experimental Data for Example Calculation of JBN Method (Ref. 53).

Injection rate, cm^3/min	3.00
Water viscosity, cp	0.81
Oil viscosity, cp	5.52
Temperature, °F	85
Core plug length, in.	3.031
Core plug diameter, in.	0.997
Pore volume, cm^3	9.67
Air permeability, md	647
Brine permeability, md	571
Initial oil saturation, fraction	0.724
Upstream pressure, psia	1,000
Confining pressure, psia	3,000
dp/q at 100% brine saturation, $psi/(cm^3/min)$	0.532
dp/q at initial oil saturation, $psi/(cm^3/min)$	5.437

Table A.3: Computed Results for Example Calculation of JBN Method.

Qi, PV	f_o	S_w	S_w	1/Qi	1/Ir	d(1/Qi)/ d(1/Qi*Ir)	k_{ro}	k_{rw}
0.000	1.000	0.276	0.276	_	1.000		1.000	0.000
0.155	0.424	0.533	0.467	6.452	0.977	0.738	0.313	0.062
0.312	0.211	0.579	0.513	3.205	0.953	0.893	0.188	0.104
0.400	0.164	0.595	0.529	2.500	0.860	0.955	0.157	0.117
0.620	0.106	0.624	0.558	1.613	0.732	1.076	0.114	0.141
0.795	0.083	0.640	0.575	1.258	0.685	1.152	0.095	0.155
0.940	0.070	0.651	0.586	1.064	0.657	1.205	0.084	0.165
1.428	0.046	0.679	0.613	0.700	0.589	1.350	0.062	0.189
2.068	0.032	0.703	0.638	0.484	0.531	1.493	0.047	0.212
3.330	0.020	0.735	0.669	0.300	0.455	1.700	0.034	0.245
5.005	0.013	0.761	0.696	0.200	0.397	1.899	0.025	0.275
10.341	0.006	0.809	0.743	0.097	0.320	2.313	0.015	0.338
20.683	0.003	0.855	0.789	0.048	0.277	2.793	0.009	0.409
41.365	0.002	0.900	0.835	0.024	0.249	3.373	0.005	0.495

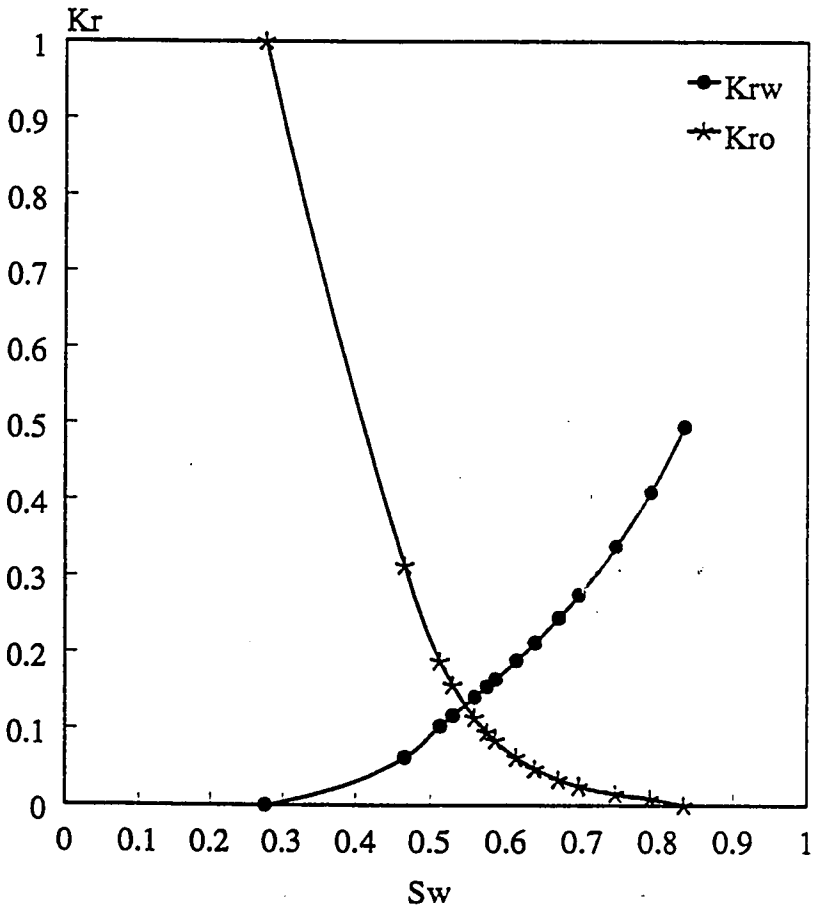


Figure A.1- Relative permeability Curves from Example Calculation of JBN Technique

NOMENCLATURE

NOMENCLATURE

A	Area, cm^2
A,B,L,M	Correlation coefficients of Chierici models
b_i	Constants
C	Constant parameter in Land's equation
E	Error or residual
Exp	Exponential function (Exp $x = e^x$)
E_{ab}	Average absolute error
E_{av}	Average error
E_{max}	Maximum absolute error
E_{min}	Minimum absolute error
F	F-test statistic
f_o	Fractional flow of oil, fraction
f_w	Fractional flow of water, fraction
I_r	Relative injectivity, fraction
k	Permeability, md
k_a	Absolute permeability, md
k_o	Effective permeability to oil, md
k_{ro}	Relative permeability to oil, fraction
k_{rw}	Relative permeability to water, fraction

$k_{ro@Swi}$	Relative permeability to oil at irreducible water saturation, frac.
$k_{rw@Sor}$	Relative permeability to water at residual oil saturation, frac.
k_w	Effective permeability to water, md
L	Length, cm
n	Number of data points
P	Pressure, psi
Q_i	Cumulative water injected, pore volume
Q_o	Cumulative oil produced, pore volume
q_o	Oil flow rate, cc/min
q_w	Water flow rate, cc/min
r	Correlation coefficient
r^2	Coefficient of determination
rms	Root mean square
S_o	Oil saturation, fraction
S_{or}	Residual oil saturation, fraction
S_w	Water saturation, fraction
S_{wf}	Water saturation at breakthrough, fraction
S_{wi}	Irreducible water saturation, fraction
$\overline{S}_w$	Average water saturation, fraction
s^2	Standard deviation
t	T-test statistic
μ_o	Viscosity to oil, cp
μ_w	Viscosity to water, cp
ϕ	Porosity, fraction
ΔP	Pressure drop, psi

Subscripts

est	Estimated value
exp	Experimental value
o	Oil phase
w	Water phase
α	Significance value

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